INTERIOR BALLISTICS

1894

GLENNON

INTERIOR BALLISTICS

WITH A SHORT TREATMENT OF THE MORE COMMON HIGH EXPLOSIVES

PREPARED AS A TEXT BOOK AND FOR PRACTICAL USE

BY

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1894

PRESS OF
DEUTSCH LITHO'G & PRINTING CO.
BALTIMORE.

PREFACE.

The methods followed in the present treatment of the subject of Interior Ballistics are an elaboration of those of a former article on "Velocities and Pressures in Guns," printed in the Proceedings of the U. S. Naval Institute in 1888 (Whole No. 45). As then stated (without proof), the so-called "general equation of motion in the bore of a gun," used by various authorities, is not perfectly general. That equation is not used in the present work, because, granting that qualification of it is necessary, simpler methods of calculating velocities and pressures in guns may be devised. There are, in fact, two conditions, only one of which can be satisfied by the equation. Looking at the subject from a mathematical point of view, if the velocity of the projectile at any point in the bore of a gun is a function of the weight of powder burned, the acceleration, and therefore the pressure at the same time, must involve the elementary change of weight, the acceleration being obtained from the expression for the velocity by differentiation; that is, the differential of the weight (in connection with the time or space differential) must appear in the expression for acceleration, and therefore, pressure. The first member of the so-claled "general equation" involves the values of the velocity and the acceleration. The second member must, therefore, involve not only the weight of powder burned, but its differential (in connection with the time or space differential), unless the quantities by which the acceleration is multiplied or divided are such as will serve to cancel the differential out. That this latter cannot be the case is evident at once from the fact that the differential varies for different shapes of

grains, in cases where the multipliers or divisors of the acceleration may be constant.

If a powder charge is completely burned at some point in the bore of a gun, the velocity at the next succeeding point differs by only an infinitesimal quantity from the value that it would have if the powder could have continued to burn. If the velocity expression involves the weight of powder, the acceleration between these two points will, in the case where the powder is supposed to continue burning, involve the rate of combustion, and in the case where no powder is burning, this rate will be replaced by zero. The so-called general equation as deduced would have all quantities except one the same in the two cases; it could not, therefore, be true for both. It is plain enough that it is true at points beyond that at which the powder ceases to burn, assuming, of course, that the gases gain equilibrium. It cannot, therefore, be true for the case where the powder continues to burn.

This leads us, by a natural process, to seek for a flaw in the deduction of the equation. As is well known, it is based on the fact that, at any point in the bore of a gun, the temperature of the expanded gas is dependent on the work done up to that point; in other words, on the velocity of the projectile. Though the gases are certainly not in equilibrium when powder is burning in a gun, an imaginary equilibrium may be assumed at any time, corresponding to such a temperature as will indicate the work done up to the time in question. An equilibrium may also be assumed corresponding to the accelerating pressure and the temperature necessary to produce it. At first sight, it would seem that these two equilibriums ought to be the same. Looking more closely into the subject, however, it will appear that there is absolutely no reason why they should be, and closer examination will prove conclusively that, unless the gas really is in equilibrium, they cannot be. The work done in a gun is dependent alone upon the expanded gases; the pressure in guns is a suitable mean of all the varying

pressures in the gun ; that is, with powder undergoing combustion it includes the pressure of the unexpanded gases as well. This is dealt with in the present book with some mathematical detail, in order to analyze, as clearly as possible, the different elements which make up the pressure in a gun. If the powder grains could be supposed so banked up against the projectile that nothing was presented at its base but unexpanded powder gas, it would not be more unreasonable to say that the resulting excessive pressure on that base was the accelerating pressure, than that, in a general distribution of the grains throughout the chamber, the accelerating pressure was that of the expanded gas alone.

The so-called general equation is true if equilibrium does exist. This would ordinarily be the case if no powder were burning; that is, at points where the weight of powder burned remained a constant. In this case, the term involving the differential of the weight of powder burned would become zero in the true general equation of motion, reducing it to the form generally used. This latter form may, however, be used if we qualify it by the supposition that, at each successive point, the weight of powder burned becomes a constant, and solve it, on that supposition, for velocity.

Simpler methods, not requiring the solution of this differential equation by the use of a series, or by the substitution of a constant in the second member (which will give exactly the same result), are available, and therefore its use is not resorted to in the present book.

The definitions and examples are taken largely from the work on Interior Ballistics by the American pioneers in this subject, Lieutenants J. F. Meigs, and R. R. Ingersoll, U. S. Navy. The general arrangement of the subject is theirs also, and the chapter on Rifling is little modified from that in their work, combined with a later paper by Lieutenant-Commander C. S. Sperry, U. S. Navy, on the same subject. Such encouragement as was necessary to an original treatment of the other parts of the subject was received from these gentlemen.

The relation between breech and projectile pressures is given, and some of the minor problems in Interior Ballistics, such as that on the recoil of a gun while the projectile is in the bore, and on the initial velocity of recoil, have been solved, and attention is called to the fact that a pressure gauge in the base of a shell does not, as ordinarily constructed, show the accelerating pressure. The methods used in finding the laws of combustion of gunpowder are different from those in ordinary use, and the law of combustion of an explosive under variable pressure is deduced on a line entirely different from anything heretofore given. It was suggested by the problem on the velocity of escape of a gas through a vent.

A chapter on smokeless powders is included. The methods by which the problem can be solved are indicated, and experience will indicate the best method to follow. It is regretted that full data for a variety of cases could not be furnished.

Nearly all the early data on the firing of guns were erratic in regard to quantities, like the density of loading, involving the volume of the powder chamber. The exact volume of the powder chamber may be calculated as readily as the inexact, and proper formulæ when the De Bange gas check is used are supplied in the present book. The data given in the examples are approximate only in many cases.

The quoted results of M. Berthelot on explosives are taken directly from the work of Messrs. Hake and McNab, who give his chemical reactions in the new chemical notation.

In a general way, the lines followed are those of M. Emile Sarrau, Ingénieur des Poudres et Saltpêtres, the eminent French writer on the action of powder in guns, and the exponent of its progressive combustion. The similarity may often be hard to trace, to one who is not a close student of the subject, but is none the less real.

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CHAPTER I.

DEFINITIONS.—POWDER CHAMBER.—FIRING TEST.

1. **Interior Ballistics.**—Interior ballistics, in general terms, may be said to include the effects produced on a projectile in the bore of a gun, and on the gun itself, when the gun and projectile are subjected to the action of the products of combustion of gunpowder or other explosives.

2. **Gunpowder Explosion.**—The phenomenon of the explosion of gunpowder may be divided into three parts, viz.: ignition, inflammation, and combustion, and these should not be confounded with one another.

IGNITION.—By ignition is understood the setting on fire of a particular part of the charge. With B. L. modern guns this is effected at the rear of the charge; and to insure ignition, a small priming charge of black powder is sometimes placed at the rear of heavy charges of cocoa powder.

INFLAMMATION.—By inflammation is meant the spreading of the fire from grain to grain throughout the whole charge. The small priming charge spoken of in the preceding definition assists in this operation in the case of cocoa powder, which ignites somewhat slowly.

COMBUSTION.—By combustion is meant the burning of *each* grain from its surface to its centre.

By *velocity of inflammation*, then, we mean the rate at which the heated and expansive gases evolved from the first grain ignited insinuate themselves into the interstices of the charge, envelop the grains, and ignite them one after another.

By *velocity of combustion* we mean the rate at which the grain burns from its surface to its centre. It is important to keep in

mind the difference of meaning of these two terms when we come to the mathematical deductions of the laws of burning of powder grains in free air.

3. The Density of Gunpowder.—The *specific gravity* of gunpowder, frequently called the *density* of the powder, is the ratio of the weight of a grain of gunpowder to the weight of an equal volume of water in standard conditions. In practice, the mean specific gravity of a number of grains must be determined. The practical limits of the density are from 1.68 to 1.90, in fact, rarely exceeding 1.85.

The density of gunpowder is denoted by δ. If v is the volume of a grain of gunpowder in cubic inches, the weight in pounds of an equal volume of water will be .03613v, since one cubic inch of water weighs .03613 pounds. Denoting the weight in pounds of the grain by w, we will have, therefore, by definition,

$$\delta = \frac{w}{.03613v} = 27.68\frac{w}{v}. \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Since nitre is very soluble in water, the practice of weighing gunpowder in mercury of known specific gravity has been universally adopted. In the instruments used, every refinement is adopted to secure precision.

4. Gravimetric Density.—The gravimetric density of gunpowder is the mean density of a suitable measure of it, interstices and all. It is usually expressed as the weight in ounces of a cubic foot of the gunpowder in grain form, the weight of a cubic foot of water being about 1000 ounces.

Generally, this gravimetric weight is from 875 to 975 ounces per cubic foot.

5. Density of Loading.—Density of loading is the ratio of the weight of a charge of gunpowder to the weight, under standard conditions, of the volume of water that would fill the powder chamber, or receptacle in which the charge is placed. It is evidently the mean density of all the products of combustion when filling the powder chamber. It is denoted by Δ.

If $\tilde{\omega}$ is the weight of charge in pounds, and C the volume of the powder chamber in cubic inches, by definition, we will have,

$$\Delta = \frac{\tilde{\omega}}{.03613C} = 27.68\frac{\tilde{\omega}}{C} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

In practice, Δ generally varies beteen .8 and 1.0. These are about one-half the values of the density of gunpowder (see Par. 3). Further along, in many practical formulæ, it is assumed that $\frac{\Delta}{\delta} = \frac{1}{2}$; which is generally a sufficient approximation.

Δ will be found tabulated for various ratios of $\tilde{\omega}$ to C at the end of the book.

6. Chamber Volume.—The powder chambers of all the larger new steel guns have the general form shown in longitudinal section in Fig. 1.

Part of the chamber is cylindrical. Its radius being r_2 and its height h_2, its volume is

$$\pi r_2^2 h_2 .$$

Part is conical. The volume of a truncated cone is equal to the sum of the volumes of three cones of the same altitude and having for bases respectively, the upper base, the lower base, and a mean proportional between the bases. Its extreme radii being r_1 and r_2 and its height h_1, the volume surrounded by the chamber slope is

$$\frac{1}{3}\pi h_1 (r_1^2 + r_2^2 + r_1 r_2) .$$

Part of the projectile extends backward into the chamber. This

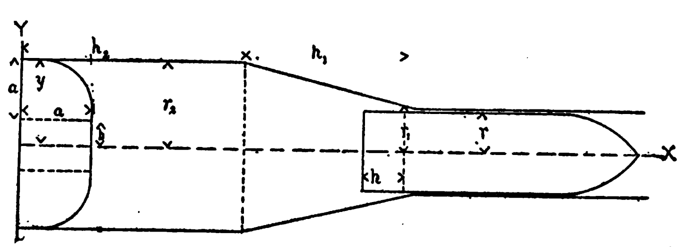


FIG. 1.

portion is subtractive. The radius of the projectile being r and the distance it extends to the rear being h, this volume is $\pi r^2 h$.

In the mushroom head, the equation to the circle of radius a is

$$x^2+(y-b)^2=a^2,$$

or,

$$y^2=b^2+a^2-x^2+2b\,(a^2-x^2)^{\frac{1}{2}}.$$

The volume of the mushroom head may be found by supposing it composed of a number of elementary cylinders of which the radius is y, and the height dx, the axes of the cylinders coinciding with the axis of the bore.

$$\therefore \text{ Volume of mushroom} = \int_0^a \pi y^2 dx$$

$$= \pi\int_0^a (b^2+a^2)\,dx - \pi\int_0^a x^2 dx + 2b\pi\int_0^a (a^2-x^2)^{\frac{1}{2}}\,dx.$$

By its form, $\int_0^a (a^2-x^2)^{\frac{1}{2}}\,dx$ is seen to be the area of a quadrant of a circle of radius a;

$$\therefore \int_0^a (a^2-x^2)^{\frac{1}{2}}\,dx = \frac{\pi a^2}{4}.$$

The remaining terms of the mushroom volume are directly integrable. We have, finally,

$$\text{Volume of mushroom head} = \pi\left(b^2a+\frac{2a^3}{3}+\frac{\pi ba^2}{2}\right).$$

This is subtractive also. The volume of the chamber, then, is

$$C=\pi\Big[r_2^2h_2+\frac{1}{3}h_1\left(r_1^2+r_2^2+r_1r_2\right)-r^2h$$
$$-\left(b^2a+\frac{2}{3}a^3+\frac{\pi}{2}a^2b\right)\Big] \quad . \quad (3)$$

7. Initial Air-Space.—The *initial air-space* in a loaded gun or shell is the portion of the chamber unoccupied by solid or liquid matter before firing. In the course of this book it will be shown (using black or brown powder) that the total volume of the solid and liquid matter practically remains the same after as before firing; a non-gaseous product, or as it is called, *residue*, equal in volume to the powder, being formed.

The volume occupied fully by charge $\tilde{\omega}$ is evidently $\frac{\tilde{\omega}}{.03613\delta}$ (compare equation (1)).

$$\therefore \text{ Volume of initial air-space} = C - \frac{\tilde{\omega}}{.03613\delta}. \quad . \quad (4)$$

8. Reduced Lengths.—In order the more readily to compare the volume of the chamber or initial air-space with that of another portion of the bore of which the length only would probably be furnished, the reduced lengths of these volumes are generally found.

The *reduced length* of any volume in the bore of a gun is the height of a right cylinder of the same volume, but with a diameter equal to the caliber of the gun.

The area of a cross section of the bore, c being the caliber, is $\frac{\pi c^2}{4}$. It is denoted by ω.

$$\therefore \quad \omega = \frac{\pi c^2}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Denoting the reduced length of the powder chamber by u_0, we have

$$u_0 = \frac{C}{\omega}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

9. Values of z.—The reduced length of the initial air-space is donated by z. In inches, then (see (4)), $\tilde{\omega}$ being in pounds, C in cubic inches, and ω in square inches,

$$z = \frac{1}{\omega}\left(C - \frac{\tilde{\omega}}{.03613\delta}\right) = \frac{1}{\omega}\left(C - 27.68\frac{\tilde{\omega}}{\delta}\right), \quad . \quad (7)$$

or,

$$z = \frac{C}{\omega}\left(1 - \frac{\tilde{\omega}}{.0361C} \times \frac{1}{\delta}\right) = \frac{C}{\omega}\left(1 - 27.68\frac{\tilde{\omega}}{C} \cdot \frac{1}{\delta}\right);$$

or, substituting the value of Δ from (2),

$$z = \frac{C}{\omega}\left(1 - \frac{\Delta}{\delta}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Also, from (7),

$$z = \frac{\tilde{\omega}}{.0361\omega}\left(\frac{.0361C}{\tilde{\omega}} - \frac{1}{\delta}\right), \quad . \quad . \quad . \quad . \quad (9)$$

and, substituting Δ from (2), as before,

$$z = \frac{\hat{\omega}}{.0361\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = 27.68\frac{\hat{\omega}}{\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right). \quad . \quad (10)$$

From (6) and (8) we have

$$z = u_0\left(1 - \frac{\Delta}{\delta}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

a most convenient form, as z will be in the same units as u_0, which may be in feet, inches, or decimeters, and the remaining quantities are abstract. Further along, approximate monomial expressions for z will be found.

The value of $\frac{z}{u_0}$ for various values of $\frac{\Delta}{\delta}$ is tabulated at the end of the book.

10. Expansions.—The travel of a projectile from its seat to any point in the bore of a gun will be denoted by u. The number of expansions of the initial air-space, corresponding to any travel of the projectile, will evidently be the volume unoccupied by solid matter, in rear of the projectile at that travel, divided by the initial air-space (see Par. 7). Denoting the number of these expansions by y, we have

$$y = \frac{\omega z + \omega u}{\omega z} = \frac{u + z}{z}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

which is used with brown and black powders, the volume of non-gaseous matter in rear of the projectile, while in the gun, remaining constant.

The expansion of the chamber volume would be used in any case where the products of combustion or explosion were entirely gaseous, as is probably true with some smokeless powders.

With such smokeless powders, in a gun, denoting the number of expansions by y_0, we have

$$y_0 = \frac{C + \omega u}{C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or, substituting the value of C from (6),

$$y_0 = \frac{u + u_0}{u_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

11. **Proof of Gunpowder.**—When gunpowder arrives at the proof-grounds for trial in the gun for which it is intended, it is subjected to examination as to its specific gravity and gravimetric density, and its pressure in the gun and muzzle velocity are ascertained with a pressure gauge and a chronoscope.

The practice of weighing gunpowder in mercury of known specific gravity has been universally adopted for finding the density. The gravimetric density is ascertained by filling a standard volume and carefully weighing the contents.

12. **Firing Test.**—Powder is tested by firing a charge in the gun for which it is intended, pressure gauges being in the breech plug, base of the shell, or elsewhere in the bore. In the WOODBRIDGE PRESSURE GAUGE, a piston with a hollow conical base threaded spirally is forced down by the powder pressure upon a copper disc, forming a truncated cone on which is traced a helix. The number of convolutions of the helix that appear on the disc shows, by comparison with discs that have been equally indented by known pressures, the pressure reached in the gun. Of course, in practice, the first step in the use of the gauge is to determine a number of points and draw a curve; the abscissæ being, say, the convolutions of the helix, and the ordinates the pressures. The other well-known pressure gauges are the RODMAN and the CRUSHER GAUGES. In the former, the pressures are ascertained by the amount of indentation of a knife in a metal cylinder, and in the latter by the shortening of a soft metal cylinder. As before, a proper table of pressures is constructed, for the various indentations or crushings, by experiment with a number of the discs in a testing machine, the results being plotted and a smooth curve drawn.

The instrument used on most proving grounds for the measurement of projectile-velocities is the LE BOULENGÉ CHRONOGRAPH. This consists essentially of a long and a short metallic rod suspended from two electro-magnets placed at different heights on a vertical standard. When the projectile passes through the first screen, the first current is broken, and the long rod or *chronometer* begins to fall. When it passes through the second screen, the second current is broken, and the short rod or *registrar* falls through a short hollow cylinder upon a trigger, releasing a knife, which, actuated

by a spring, flies forward and marks the chronometer with a horizontal line. Both currents may be broken simultaneously by the use of an instrument called the *disjunctor*. The distance of the resulting mark from the point on the chronometer abreast the knife before falling is called the *disjunction*. Captain Bréger has made some improvements upon the original Le Boulengé Chronograph. The two rods are now of exactly the same weight (though differing in size). The two electro-magnets are alike, and the strength of the current is regulated by resistance coils. The electro-magnet of the registrar may be moved at will up or down a graduated side of the vertical standard, its position being regulated by a set screw and tangent screw. The disjunction is kept constant, and proper tables for use of the chronograph are supplied, these being calculated from the formula

$$t=\sqrt{\frac{2h'}{g}}-\sqrt{\frac{2h}{g}},$$

where t is the time between screens, h the disjunction, g gravity, and h' the distance of the mark made on firing from the point of the chronometer originally abreast the knife, as indicated by a suitable measuring rule. The results found by using the tables should be corrected proportionally to $\sqrt{g}$.

For the measurement of shorter intervals of time than those ordinarily used in gun fire, that is, for use as a *micrograph*, as distinguished from a *megagraph*, the registrar and magnet are placed at the top of the standard and the registrar is dropped first. A new disjunction is found and a difference of times taken as before. There are other well known chronoscopes, such as the SCHULTZ, NOBLE and BASHFORTH. The first depends upon the fact that the number of vibrations of a tuning fork in the same temperature is constant, and the second upon the attainment of a very high velocity of a recording surface. In all these instruments electricity is used for purposes of recording.

13. **Accuracy of Crusher Gauges.**—The accuracy of crusher gauges has been examined into in a most thorough manner by Messrs. Sarrau and Vieille. Very briefly, their views are found in what follows :

Two limiting cases present themselves. In the first case, the maximum pressure in a gun or shell is the force of calibration (tarage), that is, the force corresponding, on the curve previously drawn, to the crushing or decrease of

length. In this category are included explosives of slow combustion, such as black and brown powders. The piston of the gauge is made as light as possible in order that it may start at once, its full movement developing with the pressure. In the second case, the maximum pressure is the force of calibration corresponding to half the crushing. In this category are included very quick explosions, as of the picrates and gun-cotton. The piston of the gauge is made very heavy in order that it may not start till the pressure has developed. Some explosives, as nitro-glycerine, fall under neither head, or possibly under both, occupying a mean position. A higher limit for the pressure is found, as in the first case, with a light piston, and a lower one with the heavy. These generally differ little, and in some cases have been found to agree. In the first case it is desirable to delay the explosion by using the explosive in large grains, and in the second to hasten it as much as possible by pulverization.

Examples.

1. In the two Navy rifles known as the 8″ B. L. R., Mark III, and 6″ B. L. R., Mark III, the dimensions of the powder chambers are as follows:

Gun.	r_2	a	h_2	h_1	r	r_1
8″, M. III,	4.75″	2.5″	39.46″	9.75″	3.98″	4.15″
6″, M. III,	3.5	2.0	27.84	9.65	2.98	3.15

Find the volumes of the powder chambers.

Ans. $C = 3184$ cu. in.
and 1304 "

2. The chamber dimensions of three other Navy rifles are as follows:

Gun.	r_2	a	h_2	h_1-h	r	r_1	h
6″, M. II,	3.75″	2.10″	27.08″	8.50″	2.98″	3.15″	1.65″
8″, M. II,	5.25	2.60	38.55	7.00	3.98	4.15	1.75
10″, M. I,	6.25	2.95	50.96	10.00	4.98	5.15	2.00

Required the chamber capacities. *Ans.* 1447 cu. in.
3652 "
7059 "

3. The volume of the powder chamber of an 8-inch B. L. R. is 3824 cu. in.; what is the *density of loading* when the charge is 125 pounds? *Ans.* .905.

4. What is the density of loading of the above 8-inch B. L. R. (Example 3) when the charge is 110 pounds? *Ans.* .798.

5. A 6-inch B. L. R. with a powder chamber of 1100 cu. in. capacity is loaded with a charge weighing 43 pounds; what is the density of loading? *Ans.* 1.082.

6. A 6-inch B. L. R., capacity of powder chamber 1426 cu. in., is loaded with 54 pounds of powder; what is the density of loading? *Ans.* 1.048.

7. The density of loading for the 60-pounder B. L. R. with a 10-pound charge is .8987; what is the capacity of the powder chamber? *Ans.* 308 cu. in.

8. The capacity of the powder chamber of a 6-inch B. L. R. is 1426 cubic inches; what is the initial air-space when the gun is loaded with 54 pounds of powder of density 1.818?

Ans. 603.9 cubic inches.

9. Find the reduced length of the initial air-space for the 8-inch B. L. R., capacity of powder chamber 3824 cubic inches, when loaded with 125 pounds of powder of density 1.867.

Ans. 39.212 inches.

10. A powder chamber having a capacity of 1100 cubic inches is loaded with 43 pounds of powder of $\delta = 1.75$. Find the air-space. *Ans.* $\Delta = 1.082$.

$z\omega = 420.5$ cubic inches.

11. If the charge is composed of a single grain which completely fills the powder chamber; what is the density of loading? What is it in any case if the charge just fills the powder chamber without packing? *Ans.* $\Delta = \delta$,

and $\Delta =$ gravimetric density.

12. The net volume of the chamber of a 57-mm. Hotchkiss R. F. gun is .887 litres. What are the densities of loading respectively, using .885 kilograms of black powder (C_2), .920 kilos of brown powder (brown C_2), .460 kilos of smokeless powder (BN_1), and .400 kilos of smokeless powder (BN_{144})? In French units, kilograms and litres (cubic decimetres), since a litre of water weighs a kilogram, $\Delta = \frac{\hat{\omega}}{C}$.

Ans. .998, 1.037, .519 and .451.

13. What, in the 57-mm. R. F. gun, is the number of expansions of the initial air-space to the muzzle, where $u = 20.20$ decimetres, using .920 kilos. of brown C_2 powder, given $z = 1.4$ decimetres?

Ans. $y = 15.428$.

14. What, in the above gun, is the number of expansions of the initial air-space, what the reduced length of the powder chamber and what the number of expansions of this latter, to the muzzle, when using .460 kilos. of smokeless powder (BN_1) of density 1.57? In French units, $z = \frac{\hat{\omega}}{\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$; or use Equation (11).

Ans. $y = 9.679$,
$u_0 = 3.475$ decimetres,
$y_0 = 6.81$.

15. The units used being the inch and pound, prove that in terms of the caliber,

$$z = 35.2441 \frac{\hat{\omega}}{c^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right);$$

also,

$$u_0 = 1.27324 \frac{C}{c^2}.$$

CHAPTER II.

PROPERTIES OF GASES.

14. **Specific Heats.**—The specific heat of a substance is the quantity of heat necessary to raise the temperature of unit weight of it one degree. The quantity may be measured in two ways: the body which is being heated may be allowed to expand freely under a determined pressure, or the volume of the body may be maintained constant; in the first case the specific heat is of constant pressure, and in the second, of constant volume. With perfect gases, *the specific heats under constant pressure and constant volume are independent of the pressure and volume.*

15. **Thermodynamic Laws of Gases.**—The laws connecting the pressure, volume, and temperature of a perfect gas are briefly embodied in the formula

$$pv = RxT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where p and v are its pressure and volume respectively, R a constant for the particular gas, x its weight, and T its absolute temperature; that is, its temperature measured from a zero point at a distance (in temperature) below the Centigrade zero equal to the reciprocal of the coefficient of expansion or dilatation of perfect gases. On the Centigrade scale, absolute $0°$ is at $-273°$ C. If the temperature is constant, *the pressure of a gas varies inversely as its volume.* This is MARIOTTE'S LAW. The assumption of a fixed absolute zero involves another law, namely: *the coefficient of dilatation of a perfect gas under constant pressure is constant and independent of the pressure.* This is known as GAY-LUSSAC'S LAW.

With unit weight of perfect gas (making $x = 1$ in (15)) we have

$$pv = RT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

From (16), we see:

1. If, the pressure p remaining constant, the volume varies by

the amount dv, the temperature undergoes a corresponding variation represented by $\frac{pdv}{R}$, and consequently the gas has received a quantity of heat

$$dq = \frac{c'pdv}{R},$$

c' being the specific heat of the gas under constant pressure.

2. If, the volume v remaining constant, the pressure varies by the amount dp, the temperature undergoes a corresponding change represented by $\frac{vdp}{R}$, and consequently the gas has received a quantity of heat

$$dq = \frac{c_0 vdp}{R},$$

c_0 being the specific heat under constant volume.

3. Consequently, if the volume and pressure increase together by dv and dp, the gas receives a quantity of heat

$$dq = \frac{1}{R}(c'pdv + c_0 vdp) \quad . \quad . \quad . \quad . \quad (17)$$

Differentiating (16), we have,

$$RdT = pdv + vdp, \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and, eliminating successively dp and dv between (17) and (18), we have the two equations

$$dq = c_0 dT + \frac{c' - c_0}{R} pdv, \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$dq = c'dT - \frac{c' - c_0}{R} vdp \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Substituting for p in (19), its value $\frac{RT}{v}$, from (16), we have

$$dq = c_0 dT + (c' - c_0)\frac{Tdv}{v}. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

These equations contain all the thermodynamic laws of gases.

16. **Work Done by Gas-Expansion.**—Let ψ be an infinitely small element of the surface which encloses any weight or volume

of gas. Since p is the pressure per unit of surface, the pressure on this element will be $p\psi$. If h is the displacement of this element perpendicular to itself, and the total change of volume of the gas is so small that p practically does not vary, the work done will be $hp\psi$, the work performed being measured by the product of the pressure into the distance through which it acts. The total work done on all the elements will then be

$$\Sigma hp\psi = p\Sigma h\psi.$$

But $\Sigma h\psi$ is the summation of all the elementary changes in volume, and is therefore the total change.

If, then, the total volume is increased dv, we will have

$$dv = \Sigma h\psi,$$

and the total work done will be pdv.

For finite changes of volume, where p varies, we have, by integration, for the total work denoted by ς,

$$\varsigma = \int pdv. \qquad (22)$$

17. **Mechanical Equivalent of Heat.**—Integrating both members of (19), we have

$$q = c_0 \int dT + \frac{c' - c_0}{R} \int pdv. \qquad (23)$$

If T_0 and T_1 are the original and final temperatures, (23) may be written, remembering (22),

$$q = c_0 (T_1 - T_0) + \frac{c' - c_0}{R} \varsigma, \qquad (24)$$

If the temperature of the gas remains unchanged, $T_1 = T_0$, and (24) becomes

$$q = \frac{c' - c_0}{R} \varsigma, \qquad (25)$$

or,

$$\varsigma = \frac{R}{c' - c_0} q. \qquad (26)$$

If $q = 1$,

$$\varsigma = \frac{R}{c' - c_0}. \qquad (27)$$

That is, if there is no change in temperature and one unit of heat has been absorbed by the gas, the work done must be $\frac{R}{c' - c_0}$. This, then, is the mechanical equivalent of one unit of heat, or, more briefly, the *mechanical equivalent of heat*. It is denoted by E.

$$\therefore \ E = \frac{R}{c' - c_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

E is independent of the gas used and is fixed in value. Consequently, R is proportional to the difference of the specific heats of a gas.

For perfect gases, $\frac{c'}{c_0} = 1.4$, very approximately. For other gases it varies between this limit and 1.

Placing

$$\frac{c'}{c_0} = n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

equation (28) becomes

$$E = \frac{R}{c_0(n-1)} = \frac{Rn}{c'(n-1)}; \quad . \quad . \quad . \quad . \quad (30)$$

whence, for perfect gases, we have R proportional to either specific heat of the gas.

Adiabatic Transformations.

18. Definition.—An adiabatic transformation is a change that takes place in the state of a gas within an envelope impermeable to heat; or which occurs in such a short space of time that no heat is received or lost by it.

The laws governing this change are found by placing $dq = 0$ in equations (17), (19), (20) and (21).

For adiabatic expansions, then,

$$c'pdv + c_0vdp = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$c_0dT + \frac{c' - c_0}{R} pdv = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$c'dT - \frac{c' - c_0}{R} vdp = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$c_0dT + (c' - c_0)\frac{Tdv}{v} = 0, \quad . \quad . \quad . \quad . \quad . \quad (34)$$

from which follow several important results.

19. Pressure-Volume Law.—Dividing (31) through by c_0vp, and substituting n for $\frac{c'}{c_0}$ (see (29)), we have

$$\frac{ndv}{v}+\frac{dp}{p}=0.$$

Integrating,

$$n \log v+\log p=\log k,$$

where k is some constant.

$$\therefore\ pv^n=k. \qquad (35)$$

v, the volume of unit weight, is the reciprocal of ρ, the density, or weight of unit volume.

$$\therefore\ \frac{p}{\rho^n}=k,$$

or,

$$p=k\rho^n, \qquad (36)$$

that is, *in an adiabatic change, the pressure varies proportionally to a power of the density equal to the ratio of the two specific heats.*

20. Temperature-Volume Law.—Dividing (34) through by c_0T, and substituting n for $\frac{c'}{c_0}$, as before, we have

$$\frac{dT}{T}+(n-1)\frac{dv}{v}=0;$$

k_1 being a new constant, we have on integration,

$$\log T+(n-1)\log v=\log k_1;$$

$$\therefore\ Tv^{n-1}=k_1. \qquad (37)$$

Reasoning as in the case of pressures we see that: *In an adiabatic change, the absolute temperature of a gas is proportional to a power of the density equal to the ratio of the two specific heats minus one.*

21. Pressure-Temperature Law.—Substituting for v in (33) its value $\frac{RT}{p}$ (see (16)), and dividing through by c_0T, recollecting that $n=\frac{c'}{c_0}$, we have

$$\frac{ndT}{T}-(n-1)\frac{dp}{p}=0,$$

or,

$$\frac{dT}{T}=\frac{(n-1)}{n}\frac{dp}{p};$$

k_2 being a constant, we have, by integration,

$$\log T = \log k_2 + \left(\frac{n-1}{n}\right) \log p,$$

or,
$$T = k_2 p^{\frac{n-1}{n}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

or,
$$p = \frac{1}{k_2} T^{\frac{n}{n-1}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

(38) and (39) express the relation between the temperature and the pressure of a gas in any adiabatic change.

22. **Work Done.**—Substituting for R in (32) its value $E(c' - c_0)$, from (28), and clearing of fractions, we have

$$Ec_0 dT + pdv = 0;$$
$$\therefore \quad -Ec_0 dT = pdv.$$

Integrating, remembering that the work done is (see (22)),

$$\int pdv = \varsigma,$$

and assuming T_0 and T_1 as the initial and final temperatures (compare Par. 17), we have

$$Ec_0 (T_0 - T_1) = \varsigma; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

that is, *in an adiabatic transformation the temperature of a gas is lowered by a quantity which is proportional to the external work done.*

If in (40) the final absolute temperature T_1 be made 0, we have

$$Ec_0 T_0 = \varsigma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

This is the total work that a unit weight of gas at absolute temperature T_0 is capable of doing when indefinitely expanded without gain or loss of heat.

23. **A Non-Gaseous Source of Heat.**—Suppose that we have a unit weight of permanent gas and a solid (or liquid) of weight ε inside of an envelope impenetrable by heat, and that as work is performed by gas-expansion, heat is supplied from the solid, this transfer of heat being accomplished so readily that the gas is always at the same temperature as the solid.

Denote the specific heat of the solid by c_1, the specific heats of the gas being as heretofore c' and c_0. In cooling through the infinitesimal dT, that is, in changing its temperature $-dT$, the

solid will evidently give up a quantity of heat $6c_1dT$. As the envelope is impenetrable by heat, all this is absorbed by the gas. The effect on the volume and pressure, then, is found by substituting this for dq in (17),

$$\therefore \; 6c_1dT = \frac{1}{R}(c'pdv + c_0vdp).$$

The gas, moreover, cools the same amount as the solid; that is, the change in its temperature is $-dT$. The effect of this may be found by substituting $-dT$ for dT in (18).

$$\therefore \; -RdT = pdv + vdp.$$

Eliminating RdT between these two equations, we have

$$pdv + vdp = -\frac{1}{6c_1}(cp'dv + c_0vdp);$$

transposing, and separating the variables,

$$\frac{dp}{p} + \frac{c' + 6c_1}{c_0 + 6c_1}\frac{dv}{v} = 0.$$

In this, place

$$\frac{c' + 6c_1}{c_0 + 6c_1} = n'. \qquad (42)$$

Then, integrating, denoting the constant of integration by log k, we have

$$\log p + n' \log v = \log k;$$

$$\therefore \; pv^{n'} = k, \qquad (43)$$

an equation similar to that (35) in the case of the gas alone.

It is evident that other laws similar to those already deduced for the gas alone, but involving n' in place of n, may be readily deduced. In fact, the present case is the more general, the laws for the case without the solid source of heat being derivable from it by placing $6 = 0$. The equations deduced in the course of this book for the adiabatic expansion of a gas may be made to serve the present method of expansion by substituting n' for n.

24. Work and Final Velocity.—The work done by a gas in expanding adiabatically from a volume v_0 to v_1 will evidently be (see (22) and (35))

$$\varsigma = \int_{v_0}^{v_1} pdv = k\int_{v_0}^{v_1}\frac{dv}{v^n}.$$

Integrating, remembering that $p_0 v_0^n = k$, we have

$$\varsigma = \frac{p_0 v_0}{n - 1}\left(1 - \left(\frac{v_1}{v_0}\right)^{1-n}\right). \quad . \quad . \quad . \quad (44)$$

If a number of bodies, free to move, are pushed from a state of rest by the gas in accomplishing this expansion, denoting the mass and final velocity of any one body by m and V respectively, we will have, following Par. 16,

$$\varsigma = \Sigma \frac{mV^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

If the bodies are equal in mass,

$$\varsigma = \frac{m}{2} \Sigma V^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

If, in addition, the final velocities of the bodies are equal, their number being N,

$$\varsigma = \frac{mNV^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

If only one body is moved, which assumes the mass of the remaining bodies of the system infinite, in order, according to Newton's laws, that the centre of gravity of the system may remain fixed, we have

$$\varsigma = \frac{mV^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

25. Pressure-Velocity Formula.—Denoting the number of expansions by y, we have

$$y = \frac{v_1}{v_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Substituting from (48) and (49) in (44), we have

$$\frac{mV^2}{2} = \frac{p_0 v_0}{n - 1}(1 - y^{1-n});$$

$$\therefore\ V^2 = \frac{2p_0 v_0}{m(n - 1)}(1 - y^{1-n}) \quad . \quad . \quad . \quad . \quad (50)$$

This would give in a stationary gun (neglecting the motion of the propelling agent) the velocity of a projectile, pushed by a gas, if free to move.

26. Pressure-Velocity Curve Area.—Fig. 2 represents the curves of pressure in two equal expansions of different volumes of

gas at the same original pressure. We know by the calculus that the area comprised between the curve, the axis of v and any two ordinates, as at v_0 and v_1, may be expressed by

$$\text{Area} = \int_{v_0}^{v_1} p dv,$$

and hence, by (22),

$$\text{Area} = \varsigma;$$

when a single body is moved we have, by (48),

$$\text{Area} = \frac{mV^2}{2},$$

and when a number of free bodies are moved (see (45)),

$$\text{Area} = \Sigma \frac{mV^2}{2}. \quad \ldots \ldots \quad (51)$$

These equations do not involve the law of expansion. We may then say generally :

The area of the pressure-volume curve between the original and final limits of volume is equal to the total energy gained in the same limits by the bodies impelled by the gas, supposing the bodies free to move. If the bodies are not free, the work done in overcoming their resistance to movement must be added to this energy.

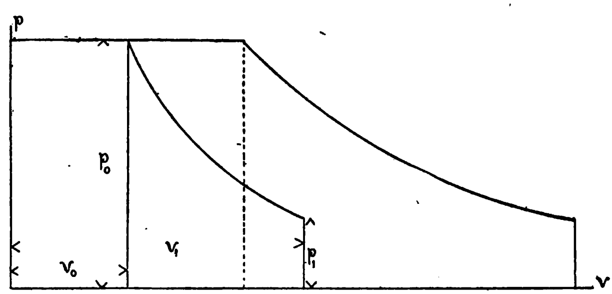


FIG. 2.

EXAMPLES.

1. Having given that the barometer is 0.7596 metre, and thermometer 18.5° C.; find the weight in kilos of a cubic metre of dry air. The weight of a cubic metre of mercury is 13,596 kilos, and

the weight of a cubic metre of air when the barometer stands at .760 and the thermometer at 0° C. is 1.2932.

See (15.) *Ans.* 1.2105 kilos.

2. Prove, in adiabatic transformations, that

$$\left(\frac{v}{v_0}\right)^{n-1} = \frac{T_0}{T} = \left(\frac{p_0}{p}\right)^{\frac{n-1}{n}}.$$

3. The volume of the powder chamber of an 8″ B. L. R. is 3824 cubic inches. This is filled with gas at a pressure of 16 tons per square inch, after which the projectile moves to the muzzle, a distance of 16.41 feet. The weight of the projectile being 250 pounds and the expansion being adiabatic, what is the muzzle velocity of the projectile, if its kinetic energy represents the total work done? Use (50), making $n = 1.4$, and $m = \frac{250}{32.2}$.

Ans. $V = 1715$ f. s.

4. The area between the curve $pv^n =$ constant, and the axis of v, taken between any finite limit and the limit $v = \infty$, is infinite if $n = 1$, and is finite if $n > 1$. What is the physical interpretation of this?

5. The muzzle velocity in a certain case is 2030 f. s. for the above 8″ B. L. R. What is the mean pressure during the travel of the shot?

$$P_M = \frac{mV^2}{2} \div \frac{u\pi c^2}{4}.$$

(Take u in feet, c in inches).

Ans. 9 tons (approx.) per sq. in.

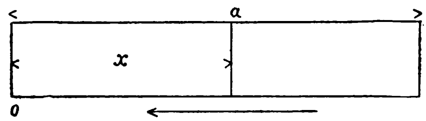
6. Determine the motion of a gun in recoiling when it is restrained by an air cylinder alone.

Since the whole recoil is performed in so short a time as to prevent the conduction outwards of any large amount of heat, the compression of the air may be assumed to be adiabatic. This gives

$$pv^n = p_0 v_0^n = p_0 (Aa)^n; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where p_0 is the initial pressure on unit area, A the effective area of

the piston, and a the length of the cylinder containing air before recoil.



Let the figure represent the air cylinder, the recoil being in the direction of the arrow. Take the origin of x at O, and take x positive to the right. Then from equation (a),

$$p = \frac{p_0 a^n}{x^n}.$$

Or, if P be the whole pressure of the air on the piston,

$$P = \frac{p_0 a^n A}{x^n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Since pressure is measured by mass times acceleration,

$$\frac{d^2x}{dt^2} = \frac{p_0 a^n A}{M} \cdot \frac{1}{x^n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

where M is the mass of the gun and carriage. From (c),

$$\int_V^{V_0} \frac{2dx \,.\, d^2x}{dt^2} = \frac{2p_0 a^n A}{M} \int_x^a \frac{dx}{x^n}; \quad . \quad . \quad . \quad . \quad (d)$$

where V_0 and V are the initial and any subsequent values of the velocity of recoil. From (d), by integration,

$$V_0^2 - V^2 = \frac{2p_0 a^n A}{M(n-1)} \left[\frac{1}{x^{n-1}} - \frac{1}{a^{n-1}} \right]. \quad . \quad . \quad . \quad (e)$$

By putting $V = 0$, and solving for x, we find the whole length of recoil. The corresponding value of P will be given by (b).

7. A quantity of air whose pressure is p_1 (much greater than the atmospheric) and absolute temperature is that of the outside atmosphere, T_0, is confined in a vessel. On opening a cock the confined air quickly expands, gaining the pressure p_0 of the outside atmosphere, when the cock is shut. No heat is absorbed or lost while the cock is open, and after shutting it, when the confined air has resumed its original temperature, its pressure is p_2. Determine

n, the ratio of the specific heats (for air). (See Theory of Heat, by Prof. J. Clerk Maxwell, tenth edition, pages 181 and 182.)

The expansion while the cock is open is supposed adiabatic. By (38),

$$\frac{p_1^{\frac{n-1}{n}}}{T_0}=\frac{p_0^{\frac{n-1}{n}}}{T_1},$$

T_1 being the absolute temperature at which the closed air arrives. By (15) (after closing the cock), the temperature rising to T_0 because of heat coming through the sides of the vessel,

$$T_1=\frac{p_0 T_0}{p_2}.$$

$$\therefore\ \frac{p_1^{\frac{n-1}{n}}}{T_0}=\frac{p_0^{\frac{n-1}{n}}p_2}{p_0 T_0}\text{; also }\left(\frac{p_1}{p_0}\right)^{n-1}=\left(\frac{p_2}{p_0}\right)^{n};$$

$$\text{also }p_1^{n-1}=\frac{p_2^{n}}{p_0}\text{; also }\left(\frac{p_1}{p_2}\right)^{n}=\frac{p_1}{p_0}.$$

$$\therefore\ n=\frac{\log p_1-\log p_0}{\log p_1-\log p_2}.$$

CHAPTER III.

EQUILIBRIUM IN AN EXPANDING GAS.

27. Time of Expansion.—If all the elements except v_0 in equation (50) are supposed constant, we will have, K^2 being a constant,

$$V^2 = K^2 v_0,$$

$$\therefore \; V = K v_0^{\frac{1}{2}}.$$

That is, the velocity of a body, if representing the sole effect of an adiabatic expansion, is proportional to the square root of the original volume of the gas. The mean velocity of the body, up to a fixed final expansion, must then be proportional to the same square root. Denoting the mean velocity by V_m, K_1 being a new constant, we have, then,

$$V_m = K_1 v_0^{\frac{1}{2}}.$$

Denoting by u the distance moved by the body for any given expansion, the surface on which the pressure acts being constant, we may write, K_2 being still another constant,

$$u = K_2 v_0,$$

and the time of expansion, which will be obtained by dividing the distance moved u by the mean velocity V_m, will therefore be

$$t = \frac{K_2}{K_1} v_0^{\frac{1}{2}}; \qquad (52)$$

that is, *in a given system, the time required for a free body, which alone is moved, to travel through the distance representing any given expansion is proportional to the square root of the original (or final) volume of the gas.* This is true for any law of expansion that does not itself depend upon the time, as may be the case where heat is lost by radiation, in other than adiabatic expansions.

If the volume of gas is infinitesimal, while the surface it acts on is finite, the gas will make a finite number of expansions in an infinitesimal time. This may be extended by the use of the summation sign to any case where an infinitesimal quantity of gas acts on a finite surface.

28. Pressure in Expanding Gas.—Assume a weight $\bar{\omega}$ of gas occupying the bore of a gun and moving, as shown in Fig. 3, a projectile of weight w, the effective accelerating pressure on the base of the projectile being P. For

simplicity, assume the volume in rear of the projectile to be *cylindrical*, its cross-section being in the ratio $\frac{1}{k}$ to that of the projectile.

Assume the cross-section of the cylinder the unit of area. Supposing that the cylinder is fixed, the pressure P_B at its base is moving not only the weight w but the entire weight $\tilde{\omega}$ of gas as well. Denote the distance between the bottom of the bore and the projectile by L_0. Let p represent the variable pressure throughout the gas and assume a cylindrical lamina of thickness dL at any distance L from the bottom of the bore. dp being the decrease of pressure in a distance dL, the lamina is subjected to a pressure of p on its forward side and of $p + dp$ on its after side. dp is therefore the pressure accelerating the lamina.

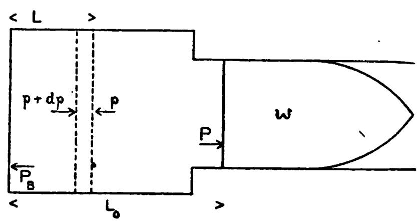


FIG. 3.

K being a suitable constant, we may write the mass of the lamina equal to $K\frac{p}{T}dL$ $\left[\text{following the law } pv = RxT \text{ (see (15)), whence } x = \frac{pv}{RT}\right]$. The constant K involves the relation between weight and mass as well as that between v, the volume of the lamina, and dL, and also includes R. Assuming the expansion ADIABATIC, we may substitute now the value of T from (38); whence we have

$$\text{Lamina Mass} = K_1 p^{\frac{1}{n}} dL, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

K_1 being a new constant.

Force is measured by the product of a mass moved and its acceleration. If a is the acceleration of the projectile, and g the acceleration of gravity, we have, then (the area of its base being k),

$$kP = \frac{w}{g}a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Place

$$\frac{L}{L_0} = \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

The acceleration of the lamina, then, is $k\gamma a$; or, substituting the value of a from (54),

$$\text{Lamina Acceleration} = P\frac{k^2\gamma g}{w}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

As before mentioned, force is measured by mass times acceleration; so by (53) and (56), substituting $L_0 d\gamma$ for dL (see (55)),

$$dp = K_1 P L_0 \frac{k^2 g}{w} p^{\frac{1}{n}} \gamma d\gamma \, . \qquad (57)$$

Separating the variables, remembering that $\gamma d\gamma = \frac{1}{2} d(\gamma^2)$,

$$p^{-\frac{1}{n}} dp = K_1 P L_0 \frac{k^2 g}{2w} d(\gamma^2) \, .$$

Integrating, noting that dp to act as a positive force must have the opposite sign as a change of p with increase of L,

$$\int_p^P p^{-\frac{1}{n}} dp = - K_1 P L_0 \frac{k^2 g}{2w} \int_\gamma^1 d(\gamma^2) \, ;$$

$$\therefore \frac{n}{n-1} \left(p^{\frac{n-1}{n}} - P^{\frac{n-1}{n}} \right) = K_1 P L_0 \frac{k^2 g}{2w} (1 - \gamma^2) \, .$$

Dividing through by $\frac{n}{n-1} P^{\frac{n-1}{n}}$, placing

$$\frac{n-1}{n} K_1 P^{\frac{1}{n}} L_0 \frac{k^2 g}{2w} = \tan^2 \theta \, , \qquad (58)$$

and transposing -1, we have

$$\left(\frac{p}{P} \right)^{\frac{n-1}{n}} = 1 + \tan^2 \theta \, (1 - \gamma^2) \, , \qquad (59)$$

or,

$$\frac{p}{P} = \left[1 + \tan^2 \theta \, (1 - \gamma^2) \right]^{\frac{n}{n-1}} ,$$

or,

$$\frac{p}{P} = \sec^{\frac{2n}{n-1}} \theta \left[1 - \gamma^2 \sin^2 \theta \right]^{\frac{n}{n-1}} . \qquad (60)$$

When $\gamma = 0$, $p = P_B$.

$$\therefore \frac{P_B}{P} = \left(1 + \tan^2 \theta \right)^{\frac{n}{n-1}} ,$$

or,

$$\frac{P_B}{P} = \sec^{\frac{2n}{n-1}} \theta \, , \qquad (61)$$

and

$$\frac{p}{P_B} = \left(1 - \gamma^2 \sin^2 \theta \right)^{\frac{n}{n-1}} .$$

(61) expresses the relation between the pressure on the projectile and that on

the bottom of the bore, but θ is as yet unknown. It is, however, so involved in the value of the mass of the gas as to be readily found.

29. Value of θ.—The mass of the gas is the sum of the masses of the laminæ of which the values are given in (53);

$$\therefore \frac{\tilde{\omega}}{g} = \int_0^{L_0} K_1 p^{\frac{1}{n}} dL = K_1 L_0 \int_0^1 p^{\frac{1}{n}} d\gamma \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Substituting the value of p from (60), and for $K_1 L_0 P^{\frac{1}{n}}$, its value from (58),

or,
$$K_1 L_0 P^{\frac{1}{n}} = \frac{n}{n-1} \tan^2\theta \frac{2w}{k^2 g},$$

and dividing through by $\frac{w}{k^2 g}$, we have

$$\frac{\tilde{\omega}}{w} k^2 = \frac{2n}{n-1} \tan^2\theta \sec^{\frac{2}{n-1}}\theta \int_0^1 \left(1 - \gamma^2 \sin^2\theta\right)^{\frac{1}{n-1}} d\gamma .$$

Place $\gamma \sin\theta = \sin\varphi$. Then $d\gamma = \frac{d \sin\varphi}{\sin\theta} = \frac{\cos\varphi d\varphi}{\sin\theta}$;

$$\therefore \frac{\tilde{\omega}}{w} k^2 = \frac{2n}{n-1} \sin\theta \sec^{\frac{2n}{n-1}}\theta \int_0^\theta \cos^{\frac{n+1}{n-1}} \varphi d\varphi . \quad . \quad . \quad . \quad (63)$$

Making $n = 1.4$,

$$\frac{\tilde{\omega}}{w} k^2 = 7 \sin\theta \sec^7\theta \int_0^\theta \cos^6 \varphi d\varphi . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

By the integral calculus,

$$\int_0^\theta \cos^6 \varphi d\varphi = \frac{1}{6} \cos^5\theta \sin\theta + \frac{5}{24} \cos^3\theta \sin\theta + \frac{15}{48} \cos\theta \sin\theta + \frac{15}{48}\theta . \quad . \quad (65)$$

$$\therefore \frac{\tilde{\omega}}{w} k^2 = 7 \sin^2\theta \sec^7\theta \left[\frac{1}{6} \cos^5\theta + \frac{5}{24} \cos^3\theta + \frac{15}{48} \cos\theta + \frac{15}{48} \frac{\theta}{\sin\theta}\right] . \quad (66)$$

In practice $\tilde{\omega}$, w and k^2 are given; θ is found by substitution and trial.

$\frac{\tilde{\omega}}{w} k^2 = .07$ when $\tan\theta = .1$; $\quad \frac{\tilde{\omega}}{w} k^2 = .49$ when $\tan\theta = .25$;

$\frac{\tilde{\omega}}{w} k^2 = .29$ when $\tan\theta = .2$; $\quad \frac{\tilde{\omega}}{w} k^2 = .73$ when $\tan\theta = .3$.

30. Illustration.—Placing n equal to 1.4 in (60) and (61), we have

$$\frac{p}{P} = \sec^7\theta (1 - \gamma^2 \sin^2\theta)^{\frac{7}{2}} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

and
$$\frac{P_B}{P} = \sec^7\theta , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

whence
$$\frac{p}{P_B} = (1 - \gamma^2 \sin^2\theta)^{\frac{7}{2}} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

A very common ratio at present between $\tilde{\omega}$ and w is $\frac{1}{2}$ or .5. If k^2 is .58 (an approximate value in the larger guns when $u = 0$, and for small travels of the projectile),

$$\frac{\tilde{\omega}}{w}k^2 = .5 \times .58 = .29;$$

$$\therefore \tan\theta = .2,$$

and $$\sec^2\theta = 1.04,$$

and $$\frac{P_B}{P} = \sec^7\theta = (1.04)^{\frac{7}{2}} = 1.147; \quad \ldots\ldots \quad (70)$$

that is, if the products in rear of the projectile were all gaseous, in the case assumed, the pressure on the face of the breech block would be one-seventh greater than that on the projectile.

In the case assumed, suppose that $k = 1$; that is, that there is no enlargement of gun chamber.

Then $$\frac{\tilde{\omega}}{w}k^2 = .5,$$

$$\therefore \tan\theta = .25 \text{ (nearly)},$$

and $$\frac{P_B}{P} = 1.236. \quad \ldots\ldots\ldots \quad (71)$$

The breech pressure would be about one-fourth greater than the projectile pressure. This would hold approximately when the projectile is at the muzzle of any gun, so loaded.*

31. **The Gun-Charge-Projectile System.**—When a gun is restrained from recoiling the highest pressure occurs at the face of the breech block, and this has been a premise in all that has preceded. Without going more deeply into the mathematics of the subject, it may be stated that similar methods of reasoning will show that if the gun recoils, the maximum pressure occurs at the centre of gravity of the system which is composed of the projectile, charge, inner and outer air (in small part), the gun and gun-carriage (and more or less, according to the firmness with which the gun is held fixed, the earth itself).

If a gun of the same weight as the projectile were suspended in a vacuum in such a way that it could recoil without resistance,—on firing, it would move as fast to the rear as the projectile does to the front, the highest pressure at any instant being at a point midway between breech face and projectile; from there the powder-gas would expand both ways.

If the gun were twice the weight of projectile, it would recoil with very approximately half the velocity of the latter; the point from which powder-gas would move both ways would be the point of rest, at approximately one-third the distance from base of bore to base of projectile, and this would be the point of maximum pressure; and so on.

32. **Momentum of Gas.**—The mass of one of the laminæ of paragraph 28 being $K_1 p^{\frac{1}{n}} dL$ (see (53)), or $K_1 L_0 p^{\frac{1}{n}} d\gamma$ (see (62)), and its velocity being $k\gamma V$,

*The great difference between the projectile and breech pressures in unchambered guns accounts partly for the excessive wave-motion in such guns.

where V is the velocity of the projectile, the momentum of the lamina (the product of mass and velocity) is $K_1 V L_0 k p^{\frac{1}{n}} \gamma d\gamma$.

Integrating, we have the total

$$\text{Momentum of the gas} = K_1 V L_0 k \int_0^1 p^{\frac{1}{n}} \gamma d\gamma. \quad . \quad . \quad . \quad . \quad (72)$$

Returning to (57) and integrating, changing the sign of dp as an increment,

$$-\int_{P_B}^{P} dp = P_B - P = K_1 P L_0 \frac{k^2 g}{w} \int_0^1 p^{\frac{1}{n}} \gamma d\gamma. \quad . \quad . \quad . \quad . \quad (73)$$

Eliminating the integral between (72) and (73), we have the

$$\text{Momentum of the gas} = \frac{w}{gk} V \left(\frac{P_B}{P} - 1 \right). \quad . \quad . \quad . \quad . \quad . \quad (74)$$

33. **Mechanical Energy of Gas.**—Mechanical energy is measured by half the product of the mass and the square of its velocity. For the expanding gas in a gun, it is the sum of those of the various laminæ ;

$$\therefore \text{ Mechanical energy of the gas} = \tfrac{1}{2} K_1 L_0 V^2 k^2 \int_0^1 \gamma^2 p^{\frac{1}{n}} d\gamma. \quad . \quad . \quad . \quad (75)$$

Dividing this by (62), member by member, and multiplying by $\frac{\bar{\omega}}{g}$, we have

$$\text{Mechanical energy of the gas} = \frac{\bar{\omega} k^2}{2g} V^2 \frac{\int_0^1 p^{\frac{1}{n}} \gamma^2 d\gamma}{\int_0^1 p^{\frac{1}{n}} d\gamma},$$

and substituting for p from (60), making $n = 1.4$, and $\gamma \sin \theta = \sin \varphi$,

$$\text{Mechanical energy of the gas} = \frac{\bar{\omega} k^2}{2g \sin^2 \theta} V^2 \left(1 - \frac{\int_0^\theta \cos^8 \varphi d\varphi}{\int_0^\theta \cos^6 \varphi d\varphi} \right). \quad . \quad . \quad (76)$$

From calculus, $$\int_\theta^\theta \cos^8 \varphi d\varphi = \frac{\cos^7 \theta \sin \theta}{8} + \tfrac{7}{8} \int_0^\theta \cos^6 \varphi d\varphi. \quad . \quad . \quad . \quad (77)$$

By (64), $$\int_0^\theta \cos^6 \varphi \delta\varphi = \frac{\bar{\omega} k^2}{7\omega \sin \theta \sec^7 \theta};$$

$$\therefore \text{ Mechanical energy of the gas} = \frac{\bar{\omega} k^2}{16 g \sin^2 \theta} V^2 \left(1 - \frac{7w \sin^2 \theta}{\bar{\omega} k^2} \right). \quad . \quad . \quad (78)$$

EXAMPLES.

1. A gun and its projectile are of the same weight, what is the ratio of the muzzle velocity when each is free to move to the muzzle velocity when the gun is held immovable, neglecting the weight of the powder?

Ans. $\frac{V_2}{V_1} = \sqrt{2}$.

2. Trace the curve of pressure throughout the gas in rear of a projectile in a gun.

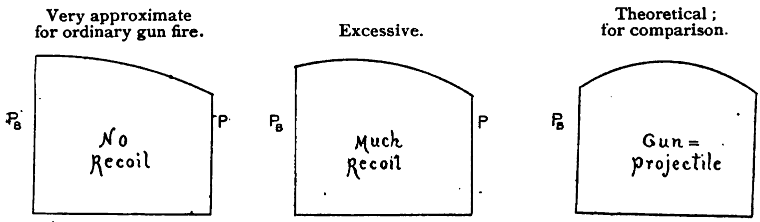


FIG. 4.

3. The products of an explosion are in a certain case all gaseous, and the maximum pressure measured with a crusher gauge in the nose of the breech block is 16 tons. If, at the instant of maximum pressure, the ratio of the actual length of the volume in rear of the projectile to its reduced length is $\sqrt{.58}$, and the weight of charge is one-half that of the projectile, what is the maximum pressure on the projectile? *Ans.* 14 tons.

4. If, in the above case, the charge had been one-eighth the weight of the projectile and the maximum pressure on the projectile had been 14 tons, what would have been the maximum recorded pressure?

$$\frac{\tilde{\omega}}{w} k^2 = .07. \qquad P_B = 14.490.$$

To interpolate for different values of $\frac{\tilde{\omega}}{w} k^2$, note that $\tan^2 \theta$ varies very approximately as $\frac{\tilde{\omega}}{w} k^2$.

5. Find, without the use of calculus, the momentum of the powder products, supposing them expanding in rear of projectile, but with all laminæ of the same density.

Assume the chamber and bore of the same diameter, or $k = 1$, and divide the volume of products transversely into N equal cylindrical laminæ. The mass of a lamina is $\frac{\tilde{\omega}}{Ng}$. The mean velocity of the parts of the first lamina (beginning at breech) is $\frac{V}{2N}$, of the second $\frac{3V}{2N}$, etc., and of the last $\frac{2N-1}{2N}V$. The total momentum will be

$$\frac{\tilde{\omega}}{Ng} V \frac{1}{2N}(1 + 3 + 5 \ldots + (2N - 1)) = \frac{\tilde{\omega}}{2g} V\left(1 - \frac{1}{N}\right),$$

and when N is infinite,

$$\text{Momentum} = \frac{\tilde{\omega}}{2g} V.$$

6. The formula for the momentum of powder-gas in terms of the maximum pressures is (74). What is the ratio of maximum breech to projectile pressure in a gun when the charge is one-half the weight of the projectile, and the momentum is that found in the last example?

We find $$\frac{P_B}{P} = 1 + \frac{\bar{\omega}}{2w}.$$ *Ans.* $P_B = 1.25\ P.$
(Compare (71).)

7. Assuming the powder products in rear of a projectile to be all at the same temperature, each lamina following Mariotte's law as to pressure, deduce an expression giving the relation between breech and projectile pressures in an unchambered gun.

e being the Napierian base,

$$p = Pe^{x^2(1-\gamma^2)} \text{ and } P_B = Pe^{x^2},$$

where x is found from

$$xe^{x^2}\int_0^x e^{-x^2}dx = \frac{\bar{\omega}}{2w}.$$

When $x = .1$,	$xe^{x^2}\int_0^x e^{-x^2}dx =$	.01007	(Evidently $\frac{\bar{\omega}}{w}$ varies approximately as x^2.)
" " .2,	"	.04108	
" " .3,	"	.09560	
" " .4,	"	.17821	
" " .5,	"	.29615	

8. Assuming, in Example 7, the charge one-third the weight of the projectile, how does the ratio $\frac{P_B}{P}$ agree with that deduced after the method of Example 6?

By Example (7),

$$\frac{\bar{\omega}}{2w} = \frac{1}{6} = .17 \quad \therefore \quad x = .4. \quad \log e = .43429 \quad \therefore \quad \frac{P_B}{P} = e^{.16} = 1.17.$$

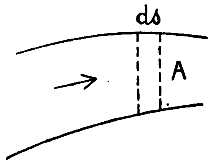
By Example (6),

$$\frac{P_B}{P} = 1 + \frac{1}{6} = 1.17.$$

9. Derive an expression giving the velocity of flow of perfect gases.

Assume a surface cutting all the stream lines of the gas at right angles, and a second surface parallel to the first and at a distance ds from it. Let $A =$ area of cutting lamina, $ds =$ its thickness, $v =$ the volume of unit weight in it, $V =$ its velocity, $dp =$ the difference of pressures on its two surfaces, and $g =$ the acceleration of gravity. Then $A \cdot ds$ is the volume of the cutting lamina, Adp

the moving force applied to it, $\frac{1}{gv}$ the mass of unit volume of gas in the lamina, and $\frac{A \cdot ds}{gv}$ the constant mass of the lamina. We have, then,



$$\frac{dV}{dt} = \frac{VdV}{dt} = \frac{d}{ds} \cdot \frac{V^2}{2} = \frac{-Adp}{\frac{Ads}{gv}} = -\frac{gvdp}{ds}.$$

10. Find the velocity of escape of gas through the vent of a chamber in which the pressure is 3000 atmospheres, the external pressure being one atmosphere.

If the phenomenon is adiabatic, we have $pv^n = p_0 v_0^n$; whence, from the result in the last example, if the velocity of the gas is zero when $p = P$, and is V when $p = p_0$,

$$V^2 = p_0 v_0 \frac{2gn}{n-1}\left[\left(\frac{P}{p_0}\right)^{\frac{n-1}{n}} - 1\right].$$

Here we have, taking metres, kilograms, and seconds as units, $p_0 = 10333$, $v_0 = \frac{1}{1.2932}$, $g = 9.81$; and also take $n = 1.4$, which is true for perfect gases.

Then $V = 2207$ metres per second, nearly.

CHAPTER IV.

PRESSURES IN A SHELL.

34. **Explosion in a Cylinder.**—If a sufficiently strong, smooth, hollow cylinder impermeable to heat were suspended horizontally in space, and in one end of it a quantity of explosive were fired, the centre of gravity of the explosive products would be transferred toward the centre of the cylinder, which would itself move therefore in the opposite direction. As the products arrived at the opposite end, they and the cylinder would gradually come to rest, and then, owing to the pressure of the gases, which would have banked up at that end, the system would take up a motion in the opposite direction. With no disturbing causes, it is easy to conceive the oscillation as thereafter continuous.

But the friction on the inside of an actual cylinder would retard the outer portions of gas, turning the regular motion of the cylindrical laminæ into the irregular movement of a number of broken portions of laminæ, the masses of gas moving together in one direction becoming smaller and smaller, till at last there would be apparently no excess of motion either way, the cylinder would have come to rest, and the gas would be in equilibrium. The cylinder will have come to rest by frictional resistance to the gas. This back resistance multiplied in each case by the distance through which it acts will be work done upon the gas, and when the cylinder has come to rest, its energy of motion will have been transferred to the gas as molecular motion or heat. The temperature of the gas will then be the same as if it had expanded to its present volume without production of work ; that is, its temperature will be that of explosion or combustion, if we suppose the actual cylinder impermeable to heat. In this case, then, if we use any two weights of explosive, the final temperature being the same, we may calculate the final pressure of one from that of the other by *Mariotte's law*. Of course, in reality, the heat produced by friction will be divided between cylinder and gas.

The above assumption of the explosion at one end is of an extremely simple case, but the irregularities spoken of exist more or less in every explosion. Not only that, but it is probably one of the irregular pressures which precedes the final, that is measured by a pressure gauge in an explosion chamber. It is doubtful if, at the end of the time required to gain equilibrium, the final pressure, diminished in actual practice by radiation of heat would equal the mean pressure of the gas at some previous period (with its lesser diminution of heat), not to speak of the highest pressure in the various parts of the disturbed gas. The records, then, of presssure gauges are, as might be expected

irregular, though the *work* done in guns is sufficiently regular. Theoretical pressures, moreover, calculated from the products and the temperature of explosion do not agree with actual pressures measured by the crusher gauge.

In what follows on pressures in shells, *Mariotte's law* is used, but with the above reservations.

35. Force of Explosives.—The force of an explosive is the pressure obtained by firing the explosive in a volume such that the mean density of the gaseous products alone will be the ratio of their weight to that of the explosive.

In decimetres, kilograms, and litres (cubic decimetres), the force of an explosive is the pressure per square decimetre developed when a kilogram is exploded in a volume such that the gaseous products alone (of the explosion) occupy a litre. The pressure, in pounds or tons per square inch at the same time, accords with the first definition.

Let $\tilde{\omega}$ be the weight in pounds of the explosive, x the weight in pounds of the gaseous products alone, C the volume in cubic inches of the explosion chamber, and v the volume in cubic inches occupied by the gases alone.

Place
$$\frac{x}{\tilde{\omega}} = \epsilon. \qquad (79)$$

Then, by definition, when (compare (2))

$$\frac{x}{.03613v} = \epsilon, \; p = f,$$

where f denotes the force of the explosive.

36. Pressure in a Shell.—From (15), we find

$$p = RT\frac{x}{v}. \qquad (80)$$

Substituting for $\frac{x}{v}$, its value, $.03613\epsilon$, when $p = f$, we have

$$f = .03613\epsilon RT. \qquad (81)$$

$$\therefore \; RT = \frac{f}{.03613\epsilon}.$$

Assuming T constant for the different explosions, and substituting for RT in (80), we have

$$p = \frac{f}{.03613\epsilon} \cdot \frac{x}{v}. \qquad (82)$$

Substituting for ϵ its value $\frac{x}{\tilde{\omega}}$, we have

$$p = \frac{f\tilde{\omega}}{.03613v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (83)$$

Let a denote the ratio of the volume occupied by the non-gaseous products at the temperature of explosion to the volume of the weight $\tilde{\omega}$ of water; that is, let a denote the volume in litres of the non-gaseous products of a kilogram of explosive.

The volume in cubic inches occupied by the non-gaseous products, then, is $\frac{a\tilde{\omega}}{.03613}$;

$$\therefore \; v = C - \frac{a\tilde{\omega}}{.03613}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (84)$$

and, substituting in (83), we have

$$p = \frac{f\tilde{\omega}}{.03613C - a\tilde{\omega}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (85)$$

Dividing numerator and denominator by $.0361C$, substituting Δ for its value $\frac{\tilde{\omega}}{.0361C}$ (see (2)), we have

$$p = \frac{f\Delta}{1 - a\Delta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (86)$$

or

$$p = \frac{f}{\Delta^{-1} - a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (87)$$

Formulæ (83), (85), (86) and (87) give the pressures in a shell on the supposition that the gaseous products of explosion do no work, their temperature being in all cases the same,—the original temperature of explosion.

The *non-gaseous products* have been variously termed the solid residue, the liquid residue, etc., in different books. In future, in the course of the present book, the term *residue* will be used to denote them.

37. **Force and Residue of Gunpowder.**—The formulæ of the last article accord very well with the results of experiments made by Messrs. Noble and Abel with black gunpowder. These eminent authorities burned pebble powder in a closed vessel, measuring the pressures by crusher gauges, the densities of loading varying by tenths from 0.1 to 1.0.

Substituting for p and Δ, in equation (86) or (87), the measured pressures and known densities of loading, we will have ten equations for the determination of the two unknown constants f and a. We may then form the two normal equations according to the method of least squares, and find the most probable values of f and a.

The values, calculated by M. Sarrau, using all the experimental results, are

$$f = 219300 \text{ kilograms per square decimetre,}$$

and

$$a = .6833.$$

The pressures, as measured by Messrs. Noble and Abel, and as calculated by M. Sarrau from (86), using these values of f and a, are given below in kilograms per square centimetre.

Density. Δ	Pressures. Measured.	Calculated.	Differences.
0.1	231	235	— 4
0.2	513	508	+ 5
0.3	839	828	+ 11
0.4	1173	1207	— 34
0.5	1684	1666	+ 18
0.6	2266	2230	+ 36
0.7	3006	2943	+ 63
0.8	3942	3869	+ 73
0.9	5112	5127	— 15
1.00	6567	6926	—359

The vessels used by Messrs. Noble and Abel were capable of holding about 1 kilogram and ½ kilogram respectively.

Practically, then, it appears that the pressures from black gunpowder, in small, closed vessels, such as shells, will be closely approximated to by using the above values of f and a in (86) or (87).

In pounds per square inch, (87) may be written, then,

$$p = \frac{31190}{\Delta^{-1} - .6833}, \quad \ldots \ldots \quad (88)$$

and in tons per square inch,

$$p = \frac{13.9}{\Delta^{-1} - .6833}, \quad \ldots \ldots \quad (89)$$

the values of f used being the equivalent of 2193 kilos. per square centimetre.

38. Value of α for Guns.—The above value of α is high, and of f low, for high densities of loading. After cooling, the volume of the residue corresponds to a value of $\alpha = .3$. The coefficient of expansion necessary to raise the value of α from .3 to .6833 notably exceeds analogous coefficients with the greater number of solid bodies. Messrs. Bunsen and Schischkoff, by what seems direct experiment, have determined, at the temperature of combustion, a value of $\alpha = .44$. The larger the chamber, the less will be the proportional loss of heat through the envelope, the density of loading remaining constant. For, the loss of heat will increase with the enclosing surface or according to the square of some linear dimension, while the quantity of heat produced will increase with the charge and enclosed volume, or according to the cube of the same dimension. The larger, then, the chamber used for the experiments, the more nearly will the measured values of f and α approach the true values. The mean of the values of $\alpha = .44$ and $\alpha = .68$ is $\alpha = .56$. The value ultimately adopted by Messrs. Noble and Abel is .57. For black powder $\delta = 1.75$, about. The reciprocal of this is $\frac{4}{7}$ or .57. That is, a kilogram of the powder occupies a volume of .57 litres, and as, returning to the definition, α is the volume in litres occupied by the residue of a kilogram of powder, it is apparent that if we choose the above mean value of α, we make the *residue equal in volume to the powder itself.* This will hereafter be found very convenient, and is therefore adopted.

M. Berthelot suggests the following formula as preferable in some respects to those already given (the value of f is derived from the experiments of Lieutenant-Colonel Sebert and Captain Hugoniot, assuming $\alpha = .55$, or the reciprocal of the density):

$$p = \frac{4030}{\Delta^{-1} - .55}, \qquad \qquad (90)$$

the units being the kilogram and square centimetre; and Messrs. Noble and Abel give as their final formula in tons per square inch,

$$p = 18.49 \frac{\Delta}{1 - .57\Delta}, \qquad \qquad (91)$$

or,

$$p = \frac{18.49}{\Delta^{-1} - .57}. \qquad \qquad (92)$$

These formulæ are probably fairly accurate for large explosion-chambers, such as the chambers of large guns; it being remembered that they cease to apply when work is done. There are no corresponding formulæ for brown powders, but the accuracy with which the velocity formulæ hereafter deduced for black powders apply to brown powders indicates that for them also, as a close approximation, the *volume of the residue equals that of the powder.*

In the general case, then, for black or brown gunpowder in guns, we place $\alpha = \frac{1}{\delta} = \delta^{-1}$ in (87), and we have

$$p = \frac{f}{\Delta^{-1} - \delta^{-1}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

or, remembering that v in (83) becomes the initial air space or ωz,

$$p = \frac{f\tilde{\omega}}{.03613\omega z} = 27.68\frac{f\tilde{\omega}}{\omega z}. \quad . \quad . \quad . \quad . \quad (94)$$

39. **Theoretical Pressures.**—Another value of f may be readily found. After explosion in a closed vessel, the products are allowed to cool, the quantity of heat being measured by a calorimeter. This quantity divided by the product of the weight and mean specific heat (under constant volume) of all the products of explosion will give the difference in temperature, which added to the final absolute temperature (after cooling) will give the absolute temperature of combustion. In calculating f by (81), this last will be T.

The capacity of the explosion vessel is supposed to be known beforehand. If not, it may be measured, after cooling, along with p and x. Subtract the volume of the liquid and solid products from that of the chamber; the result will be the volume of gaseous products, which substitute for v in equation (80). The weight of the powder minus that of the residue will be x in the same equation. The values of the pressure and the absolute temperature after cooling are also substituted in (80) for p and T. Equation (80) will then furnish the value of R. Equation (79) will give the value of ϵ, and the values of ϵ, R and the absolute temperature of combustion will give the value of f, when substituted in (81). This method supposes the products in the same state during explosion as after cooling; if not, the change in state of any part may be compensated by a proper change in the specific heat, and by remembering that the heat of liquefaction or vaporization must be omitted in calculating the temperature of combustion. Chemical reactions during the process of cooling likewise engage heat, but known changes may be compensated as in the change of state of the same substance.

With products in the same state, the mean specific heat varies, increases probably, with the temperature, so that the absolute temperature of combustion cannot be calculated with any great degree of accuracy.

Direct measurement of the heat by a calorimeter is generally unnecessary, except for verification, as the quantities of heat engaged or disengaged by known chemical changes have already been so fully calculated and tabulated. The densities are likewise known, and from those for the separate gases R may be found for the mixed gas.

Assuming that the explosion of black gunpowder may be represented by the reaction

$$2KNO_3 + S + 2C = K_2S + 3CO_2 + 2N,$$

M. Berthelot calculates for that powder, in atmospheres, $f = 4592$, $\alpha = .12$; and, substituting the value of f in (87),

$$\text{Theoretical pressure} = \frac{4592}{\Delta^{-1} - .12};$$

or, assuming the potassium sulphide as vaporized during combustion (it vaporizes at a comparatively low temperature), $f = 5740$, $\alpha = 0$, and

$$\text{Theoretical pressure} = 5740\Delta.$$

The chemical reaction that takes place is really more intricate.

Taking that of Dr. Debus as probably the best single one,

$$16KNO_3 + 21C + 7S = 13CO_2 + 3CO + 5K_2CO_3 + K_2SO_4 + 2K_2S_3 + 16N,$$

M. Berthelot calculates (in atmospheres),

$$\text{Theoretical pressure} = \frac{4350}{\Delta^{-1} - .26}.$$

It seems, according to Dr. Debus, that any increase in the gaseous products of a gunpowder explosion is accompanied by a corresponding decrease in the temperature of combustion, so that the force of a given powder is nearly constant.

The products of explosion, with nearly all explosives, probably differ from those found after cooling.

In the succeeding pages, the theoretical pressures are denoted by $\overline{TP}$. In all cases they are given as calculated by M. Berthelot. They furnish a useful method of comparison.

40. **Detonation.**—Detonation is exceedingly quick explosion. It seems, unlike ordinary combustion, to be independent of the pressure of the surrounding medium, and to be propagated as "a wave requiring at the beginning an amount of heat or energy greater than that which is indispensable for the propagation of the wave." The velocity of this explosive wave has been measured by placing the explosive in a long tube, and detonating it at one end, the flame breaking an electric current at intervals as it proceeds along the tube, the time between the breaks being recorded by a chronograph. Detonation is never absolutely instantaneous. M. Berthelot makes a mean

measurement of this wave in gun-cotton of 5200 metres per second, in nitro-glycerin of 1000 to 1400 metres per second, and in dynamite of double this latter. The velocity varies, however, with the size of the confining tube and with other conditions.

An explosion by combustion, where the velocity of combustion depends upon the pressure of the surrounding medium, or, in other words, is due to the continuous shock of impact on the surface of the explosive by the surrounding gaseous molecules, is sometimes called an *explosion of the second order*; as distinguished from this, is the shock transmitted through the whole body of the explosive by a sufficiently heavy original blow, causing a detonation, or *explosion of the first order.*

There are also intermediate explosions, partial detonations followed by combustion, and *vice versa.*

41. **Detonated Gun-Cotton.**—Messrs. Sarrau and Vieille, by means of crusher gauges, measured the following pressures when gun-cotton was detonated in a closed space. The experiments were made before the existence of the present lights on the subject of crusher gauges, but probably represent the first shattering effect of the explosion, in shell-bursting, with all desirable accuracy. The pressures are in kilograms per square centimetre.

Δ	p
.10	1185
.15	2205
.20	3120
.30	5575
.35	7730
.45	9760
.55	11440

Detonated in its own volume, gun-cotton actually gives, according to M. Berthelot, a pressure of 9825 kgms. per square centimetre.

For high densities of loading, the chemical reaction approximates to

$$2[C_{24}H_{18}(NO_3H)_{11}O_9] = 24CO + 24CO_2 + 12H_2O + 17H_2 + 11N_2,$$

for which, in atmospheres,

$$\overline{TP} = 16750\Delta.$$

The products are different for different conditions, and vary with the density of loading. For wet gun-cotton containing 10 per cent. of added water, the above theoretical pressure is diminished by one-third, and with 20 per cent., by nearly one-half.

M. Berthelot says of gun-cotton, "it is especially distinguished by the magnitude of the initial pressures." . . . "Pure nitro-glycerin, weight for weight, realizes a work greater by one-half than gun-cotton, the initial pressure being the same."

42. **Dynamite, Nitro-Glycerin, Explosive Gelatin and Nitro-Mannite.**—The pressures caused by detonating dynamite containing 75 per cent. by

weight of nitro-glycerin in a closed space were observed by Messrs. Sarrau and Vieille before the present lights on crusher gauges. The results probably represent their effects as bursting charges in shells with sufficient accuracy. The pressures are given in kilograms per square centimetre:

DYNAMITE (75 PER CENT.).

Δ	p
.2	1420
.3	2890
.4	4265 (3894 and 4546)
.5	6724 (6902 and 6546)
.6	9004.

According to new trials, with a very heavy piston in the crusher gauge, for a density of loading of .3, the pressure was 2413 kgms.

M. Berthelot uses these results to calculate by Mariotte's law the pressure in case the silica (estimated at 0.1 cc. for 1 grm. of dynamite) were removed. The results are as follows, in kilograms per square centimetre:

NITRO-GLYCERIN.

Δ	$p\Delta^{-1}$
.154	9230
.233	12430
.313	13640
.400	16800
.476	18900

Nitro-glycerin, exploded in its own volume, actually gives, according to M. Berthelot, a pressure of 12376 kgms. per sq. cm.

In atmospheres, for dynamite (75 per cent.), estimating the specific heat of silica as constant and equal to .19,

$$\overline{TP} = 14759\,\Delta\,; \text{ and in kilograms, } \overline{TP} = 15281 \text{ per sq. cm.}$$

The explosion of nitro-glycerin corresponds to a very simple reaction,

$$2C_3H_2(NO_3H)_3 = 6CO_2 + 5H_2O + 6N + O.$$

For this reaction, in atmospheres and kilograms per square centimetre respectively,

$$\overline{TP} = 18966\Delta \text{ and } 19580\Delta.$$

For dynamite with a nitro-cellulose base, variously called explosive gelatin, blasting gelatin, or gum dynamite, the reaction of explosion is approximately

$$51C_3H_2(NO_3H)_3 + C_{24}H_{22}(NO_3H)_9O_{11} = 177CO_2 + 143H_2O + 81N_2,$$

a lower order of nitro-cellulose, collodion-cotton being used. For this reaction, in atmospheres,

$$\overline{TP} = 19220\Delta.$$

The specific gravity or density of both nitro-glycerin and explosive gelatin is 1.6.

For nitro-mannite, the reaction being

$$C_6H_2(NO_3H)_6 = 6CO_2 + 4H_2O + 3N_2 + O_2,$$

$$\overline{TP} = 17220\Delta \text{ atms, or } 17760\Delta \text{ kgms. per sq. cm.}$$

Its apparent density is also 1.6.

43. Fulminate of Mercury.—The reaction being

$$CHg(NO_2)CN = 2CO + N_2 + Hg;$$

in atmospheres,

$$\overline{TP} = 6280\Delta.$$

Experiments made by M. Berthelot and M. Vieille gave the following results (uncorrected) in kilograms per square centimetre:

Δ	p
.1	480
.2	1730
.3	2700

Reduced in accordance with a more exact estimation of the force of calibration, the pressure agrees very closely with that of theory; it is

$$p = 6200\Delta.$$

At a density of loading of 4.43, that is, when $\Delta = \delta$, and the fulminate explodes in its own volume,

$$p = 27470$$

kilograms per square centimetre. Nitro-glycerin under the same circumstances only gives 12376 kgms., while gun-cotton gives 9825 kgms. The immensity of this pressure, combined with its sudden development, explains the part played by mercuric fulminate as priming.

44. Detonation and Combustion.—Messrs. Roux and Sarrau found that the necessary charges for breaking a certain shell varied in the inverse ratio of the following numbers, the values of which are calculated by taking that of gunpowder as a unit.

	Detonation.	Combustion.
Nitro-glycerin,	10.0	4.8
Gun-cotton,	6.5	3.0
Picric acid,	5.5	2.0
Potassium picrate,	5.3	1 8

The weight of the bursting charge of black gunpowder itself under the influence of nitro-glycerin primed with mercuric fulminate has been reduced in the proportion of 4.34 to 1.

The inequality in the destructive effect of the same explosive caused by the method of ignition is attributable, first, to the greater loss of heat in the

slower explosion; second, to a different chemical reaction, there being more dissociation in the more violent reaction, and third, to mechanical effects; though the mean pressure throughout the products were equal in two cases, the greater inequality in pressure in the parts of the same gas would in the more sudden explosion produce higher local pressures, thereby causing greater disruptive effect.

EXAMPLES.

1. The tensile strength of forged steel is assumed to be 100,000 pounds per square inch; what charge of powder is necessary to just rupture the walls of the 6-inch B. L. R. steel shell, the capacity of the cavity being assumed to be 50 cubic inches, and the internal radius of the cavity 1.5 inches?

The internal pressure that a simple cylinder can safely stand is given by

$$p = \frac{3\theta(R_1^2 - R_2^2)}{4R_1^2 + R_0^2},$$

where θ is the strength of metal. The safe pressure in the above shell is therefore $p = 52,940$ lbs. per square inch. From (88),

$$\Delta^{-1} = \frac{1}{\Delta} = .6833 + \frac{31190}{p};$$

$$\therefore \ \Delta^{-1} = 1.2725; \text{ by (2), } \tilde{\omega} = .03613\,\Delta C,$$

$$\therefore \ \tilde{\omega} = \frac{.03613 \times 50}{1.2725} = 1.42 \text{ pounds. } \textit{Answer.}$$

2. What weight of 75 per cent. dynamite will rupture the walls of the shell in Example 1?

(See Table, Paragraph 42.)

1 kilogram $= 2.2046$ lbs., 1 square decimetre $= 15.5$ square inches. Consequently, to convert pounds per square inch into kilograms per square decimetre, multiply by $\frac{15.5}{2.2046} = 7.03$.

Ans. $p = 372168$ kgms. per sq. dec.
$\Delta = .36$,
$\tilde{\omega} = .65$ lb.

3. Suppose the strength of the walls of a 6″ B. L. R. to be 30 tons per square inch, how much gun-cotton confined in a space of 55 cubic inches in the bore will just suffice to destroy the gun?

(Use Table, Paragraph 41.)

To convert tons per square inch into kilograms per square decimetre, multiply by 15747.

Ans. $\Delta = .27$,
$\tilde{\omega} = .54$ lb.

4. An 8″ B. L. R. has a chamber capacity of 3824 cubic inches. In loading, the projectile is pushed a certain distance u forward of the compression slope, and is there supposed firmly wedged. The charge is 125 lbs. On firing, a pressure of 15.4 tons is developed. Required the value of u, using (92).

(Solve (92) for Δ^{-1}.) Equation (2) may be written

$$C = 27.68\tilde{\omega}\Delta^{-1}$$

(where C is the volume now in rear of projectile). Also,

$$C = 3824 + \frac{\pi}{4}c^2u.$$

Ans. $\Delta^{-1} = 1.77$.
$C = 6124$ cu. in.
$u = \dfrac{3300}{50.2656} = 45.75''$.

5. The volume of the bore of the above gun is 13732 cubic inches. Supposing the muzzle closed, what pressure would be developed by firing 125 pounds of powder? (Use (92).) (2) may be written,

$$\Delta^{-1} = \frac{.03613C}{\tilde{\omega}}.$$

Ans. $\Delta^{-1} = 3.97$
$p = 5.44$ tons per sq. inch.

6. A sphere of gunpowder weighing one gram is put in a cubic centimetre and fired. It being assumed that the walls of the envelope are unyielding, find in how long a time it will be completely consumed, if the speed of burning follows M. Sarrau's law.

$$V = V_0\left(\frac{p}{p_0}\right)^a. \quad \ldots \quad (a)$$

Let ρ be the weight of the powder per unit volume, w the weight burned after any time from the origin, R its primitive, and r any subsequent radius, then

$$\mathrm{w} = \frac{4\pi\rho}{3}(R^3 - r^3). \quad \ldots \quad (b)$$

Let the initial pressure in the cube be one atmosphere (p_0); then, as the sphere burns, the pressure will depend upon the weight burned alone (the residue being equal in volume to the powder), following the law

$$p = p_0 + k\mathrm{w};$$

or, placing

$$f' = \frac{f\delta}{\delta - 1},$$

$$p = p_0 + (f' - p_0)\mathrm{w}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (c)$$

since when $\mathrm{w} = 1$, $p = f'$. Therefore, inserting the value of w in (c), and substituting in (a), we have

$$\frac{dr}{dt} = -V = -V_0 \left[\frac{p_0 + (f' - p_0) \frac{4\pi\rho}{3} (R^3 - r^3)}{p_0} \right]^{a}. \qquad (d)$$

This gives the solution of the problem. For its numerical solution, taking centimetres, grams, and seconds as units, we have

$$\frac{4\pi\delta}{3} R^3 = 1;$$

whence, if we make $R = \frac{1}{2}$, so that the sphere will just go in, $\delta = 1.9098$. When expressed in atmospheres, we have $p_0 = 1$, $f' = 6333$ (Messrs. Noble and Abel's experiments). M. Sarrau takes $a = \frac{1}{2}$; and $V_0 = 1$ cent. is a fair value. Making these substitutions, we have, from (d),

$$\frac{dr}{dt} = -79.58\,(1 - 8r^3)^{\frac{1}{2}}.$$

Developing the expression under the radical by the binomial theorem, and retaining 5 terms, we find, calling T the whole time of burning: $T = .0077$ sec.

7. From Example 6, derive the law connecting the time with the radius of the portion remaining of a sphere, when burned in a closed cylinder, in a density of loading of unity.

Retaining three terms only and taking the density as before, we have

$$t = \frac{1}{79.58}\left[R - r + (R^4 - r^4) + \frac{24}{7}(R^7 - r^7) \right].$$

CHAPTER V.

QUICK POWDERS IN GUNS.

45. Theories on the Expansion of Powder-Gas.—There are two principal theories on the expansion of powder-gases in guns: First, that the gases expand adiabatically; second, that the gases receive heat from, and remain at the same temperature as the non-gaseous residue. The first is followed by M. Sarrau, the second by Messrs. Noble and Abel. The first has some advantage,—the gun is heated; heat is therefore lost to the gun from the products of combustion. If, now, the residue supplies to the gases as much heat as they supply to the gun (and with our present knowledge it would be difficult, if not impossible, to decide which would supply the greater quantity), or better, if we suppose, as an equivalent in place of this, that the residue alone communicates all the heat lost directly to the gun, the first theory is correct. If the gases render to the gun more heat than they receive from the residue, neither theory is absolutely correct, but the first is very much more approximate than the second. If the residue communicates to the gases more heat than they communicate to the gun, one is probably equally close with the other. The first is the one here adopted.

46. The Value of n.—The value of $n = \frac{c'}{c_0}$ (see (29)), the exponent of v in adiabatic transformations, is not known with certainty for powder gases. For perfect gases, its value is about 1.4. Very highly heated gases approach the condition of perfect gases, and in detonation may possibly go beyond it. In fact, M. Sarrau suggests in one place as an explanation of the phenomenon of detonation, that the dissociation of the parts may be carried to the atoms of the molecules, the gases becoming mono-atomic; in this case, n would have a theoretical value of $\frac{5}{3}$. In that event, the value of $R = Ec_0(n - 1)$ (see (30)) would be nearly doubled, and therefore the value of p (see (15)).

The value of n, used in the present treatise, is that for perfect gases, or

$$n = 1.4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (95)$$

47. Pressures Using Instantaneous Powders.—The pressure caused by burning a weight of powder, $\tilde{\omega}$, in the chamber of a gun may be found, if the combustion is complete before the projectile has begun to move, by (94). Denoting the chamber pressure by p_1, we have, then,

$$p_1 = \frac{f\tilde{\omega}}{.0361\,\omega z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

As the projectile moves along the bore of the gun, the pressure decreases, following the law $pv^n = k$ (see (35)), all the changes in the gas being supposed adiabatic. When the projectile has traveled a distance u, the volume occupied by the gases is $\omega(z+u)$. Denoting the pressure at this point by p, we have, by (35),

$$p_1\,(\omega z)^n = p\,[\omega\,(u+z)]^n.$$

Substituting the value of p_1 from (96), and solving,

$$p = \frac{f\tilde{\omega}}{.0361\,\omega z}\left(\frac{z}{u+z}\right)^n;$$

or, placing $\dfrac{u+z}{z} = y$, the *number of expansions* of the initial air-space (see 12)),

$$p = \frac{f\tilde{\omega}}{.0361\,\omega z}\,y^{-n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

Equation (97) is the expression for the pressure (per square inch) that would exist in the bore of the gun in case the powder gases in rear of the projectile could gain equilibrium in the sense of having the pressure at breech and projectile the same; p is less than the pressure on the breech block and greater than that on the projectile, which in turn is greater than the effective accelerating pressure (see Chapter III). We will denote the effective accelerating pressure per square inch (the pressure which is measured by the forward motion of the projectile) by P, and its ratio to p by i; whence we have

$$P = ip, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

where i is less than unity. From (97) and (98),

$$P = \frac{if\tilde{\omega}}{.0361\,\omega z}\,y^{-n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

In (97) and (99), f and $\tilde{\omega}$ are taken in pounds and ωz in cubic inches. P will be in the same units as f.

48. **Velocities with Instantaneous Powders.**— The total accelerating pressure acting on the base of the projectile is ωP.

From (99), we have

$$\omega P = \frac{if\tilde{\omega}}{.0361z} y^{-n}. \quad . \quad . \quad . \quad . \quad . \quad (100)$$

We know, from mechanics, that *force* is measured by the product of *acceleration* and *mass* accelerated. The acceleration of the projectile, V being its velocity, is $\frac{dV}{dt}$, and its mass, w being its weight, is $\frac{w}{g}$;

$$\therefore \ \omega P = \frac{w}{g}\frac{dV}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

From (100) and (101), we have

$$\frac{dV}{dt} = \frac{if\tilde{\omega}g}{.0361zw} y^{-n}. \quad . \quad . \quad . \quad . \quad . \quad (102)$$

Now,
$$\frac{dV}{dt} = \frac{2VdV}{2Vdt} = \frac{d(V^2)}{2Vdt},$$

and
$$V = \frac{du}{dt}, \quad \therefore \ Vdt = du;$$

whence,
$$\frac{dV}{dt} = \frac{d(V^2)}{2du}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Also,
$$\frac{u+z}{z} = y \text{ (see (12))};$$

$$\therefore \ u + z = zy,$$

and, remembering that z is a constant, we have

$$du = zdy.$$

(103) then becomes

$$\frac{dV}{dt} = \frac{d(V^2)}{2zdy}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

From (102) and (104), we have, then,

$$\frac{d(V^2)}{2dy} = \frac{if\tilde{\omega}g}{.0361w} y^{-n},$$

or, $$d(V^2) = \frac{if\tilde{\omega}g}{.0181w} y^{-n}\,dy.$$

Integrating both members of this equation between corresponding limits, we have

$$\int_0^V d(V^2) = \frac{if\tilde{\omega}g}{.0181w}\int_1^v y^{-n}\,dy;$$

$$\therefore\ V^2 = \frac{if\tilde{\omega}g}{.0181w(n-1)}(1 - y^{1-n}). \quad . \quad . \quad (105)$$

Substituting for n its value 1.4, placing

$$1 - y^{-.4} = Y, \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

and $$\frac{g}{.00723} = H,$$

we have $$V^2 = Hif\frac{\tilde{\omega}}{w}Y. \quad . \quad . \quad . \quad . \quad . \quad (107)$$

The value of the logarithm of Y is tabulated at the end of the book. For the same powder in different guns, f is constant, and if we assume i a constant for the same powder, placing, in (107),

$$Hif = H_1,$$

we will have, for the same powder in different guns,

$$V^2 = H_1\frac{\tilde{\omega}}{w}Y. \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

For different travels in the same firing of a gun, placing

$$H_1\frac{\tilde{\omega}}{w} = H_2,$$

we have $$V^2 = H_2Y. \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

49. **General Expression for Pressure.**—Suppose a portion of the charge to be burned instantaneously in a gun, producing a weight of gas which we will denote by x. This gas makes a certain number of expansions which we denote by y. An infinitesimal thickness on the exterior of what remains of the powder grains is then burned. We will denote the total weight of the gas produced by these external layers by dx. Now, the weight of gas dx has at its formation a certain fixed, though unknown, volume, independent of the pressure of the surrounding medium, a certain temperature, that of combustion of the powder, and a pressure depending on its weight, volume, and temperature (see (15)). In short, the state of the gas at its instant of formation is fixed

and definite. This pressure at formation is much greater than that of the expanded gas of the weight x.

Following the reasoning of Paragraph 27, it is apparent that, as the infinitesimal weight of gas dx acts inside of a finite surface, namely, that remaining to the powder grains, it can be expanded to equilibrium in an infinitesimal time. The pressure throughout the gas of weight dx will then start with a value much higher than that throughout the gas of weight x, and will end with the same. The mean pressure of the gas dx during this expansion will, then, be higher than that of the gas x.

Denote the expansion in volume of gas x in the infinitesimal time dt by dv'; denote the expansion in volume of gas dx in the infinitesimal time dt by dv''. The total change of volume, then, is $dv = dv' + dv''$.

Denote the pressure of gas x by p', and denote the mean pressure of gas dx expanding during dt by p''. Then the work done by the expansion of gas x during time dt is $p'dv'$, and the work done by the expansion of gas dx during time dt is $p''dv''$. The total work done by the gas is, therefore, $p'dv' + p''dv''$. This, divided by the change of volume $dv' + dv''$, will give the mean pressure throughout the whole gas;

$$\therefore\ p = \frac{p'dv' + p''dv''}{dv' + dv''} = \frac{p'dv' + p''dv''}{dv}. \quad . \quad . \quad . \qquad (110)$$

The ratio i will furnish the accelerating pressure as in (98), and we will have

$$P = i\,\frac{(p'dv'' + p''dv'')}{dv' + dv''} = i\,\frac{p'dv' + p''dv''}{dv}. \quad . \quad . \quad . \qquad (111)$$

In (111), p'' is always greater than p'. The ratio of dv'' to dv' may always be calculated, so that (111) can be used to calculate the accelerating pressure. The ratio mentioned is finite, dv' being due to the infinitesimal expansion of a finite volume, while dv'' is due to the finite expansion (finite number of expansions) of an infinitesimal volume. As p'' is always greater than p', it is apparent, during the combustion of the charge, that P is always greater than ip'. We may arrive at the same value of P as that in (111) in another and preferable way, as will appear later on.

50. Work Done by Powder.—When equilibrium is established, the gas dx has expanded, and added to the system a certain amount of mechanical energy which we have denoted by $p''dv''$. Now, all the powder gas is at its creation in the same state. In equilibrium, the parts are again in the same state. All the various parts have gone through the same changes, and the expression representing the work done by a weight dx taken from any portion of the gas x must be the same as that already deduced for the part dx when burned by itself, or $p''dv''$. All are, therefore, in the same state as if they had burned instantaneously, and had expanded to the present volume. The above reasoning may be continued with any number of infinitesimal additions, and thus to the progressive burning of powder in guns.

It appears, then, whether a given weight of powder is burned instantaneously or progressively that the work done throughout the system, or the mechanical energy added to it, is the same.

51. Work of the System.—Let us denote by V_1, the velocity acquired by any mass m forming part of the gun-charge-projectile system. Denote by R, the resistance offered to the free motion of any part of the system through distance U. The total work done by the interior powder gas will then be (compare with Paragraphs 16 and 24),

$$\varsigma = \Sigma \frac{mV_1^2}{2} + \Sigma RU;$$

p, the pressure when the gas is in perfect equilibrium, is evidently the equivalent of the varying pressure which, distributed throughout the powder gas, does the work of the system. We may then write,

$$\int pdv = \Sigma \frac{mV_1^2}{2} + \Sigma RU.$$

Similarly, the effective pressure P accounts entirely for the mechanical energy of the projectile, and we may write,

$$\int Pdv = \frac{wV^2}{2g}.$$

Since $P = ip$ (see (98)), we have

$$\frac{wV^2}{2g} = i\left(\Sigma \frac{mV_1^2}{2} + \Sigma RU\right);$$

or,

$$\Sigma \frac{mV_1^2}{2} + \Sigma RU = \frac{wV^2}{2ig} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

The velocities of the parts of the system are connected by a fixed, invariable law, namely, the centre of gravity of the parts taken as a whole remains constant. On the supposition, in any two explosions, that the resistances are the same, as well as the total work, the velocities of the several parts respectively are equal, and i is the same for the two explosions.

52. Muzzle Velocity and Muzzle Pressure with Quick Powders.—Quick powders, for any given conditions of loading and firing, are powders which are completely burned before the projectile leaves the muzzle of the gun. The muzzle velocity, if all resistances remain the same, is evidently the same for quick as for instantaneous powders. We may, then, without error, write for the square of the muzzle velocity, using quick powders, either of the three formulæ (107), (108) and (109), V being found, for the

given expansion of the initial air-space, from Table I. If, in (99), we place

$$\omega = \frac{\pi c^2}{4},$$

and substitute for if its value (see Paragraph 48) for any one condition of loading in the same gun,—that is, place

$$if = \frac{H_1}{H} = .00723 \frac{H_1}{g} = .00723 H_2 \frac{w}{\tilde{\omega} g},$$

and if we make

$$\frac{dY}{dy} = .4y^{-1.4} = X, \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

we find

$$P = \frac{2w}{\pi c^2 z} H_2 X. \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

This is the pressure in a gun at points where the powder is completely burned. In (114), c is taken according to the unit used for the pressure, generally in inches; and z is in the units of the velocity formula, generally feet; log X is tabulated in Table II.

53. **Velocities in Guns.**—Slow powders are those which are not entirely burned when the projectile leaves the muzzle. If $\tilde{\omega}\theta(y)$ represent the weight of powder burned at any expansion y, we may evidently write for all powders, and all guns, at all travels, following (107),

$$V^2 = Hif \frac{\tilde{\omega}}{w} Y\theta(y). \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

54. **Gun-Fire Reversed.**—The way in which powder acts in guns may possibly be rendered more clear by studying its opposite. Suppose, then, that we could stop the phenomenon of gun-fire just as the base of the projectile leaves the muzzle of the gun. Allow the powder gas to gain equilibrium, then push the projectile back down the bore compressing the powder gases, and suppose the products distributed chemically as at the instant of their formation. When the powder gases are compressed to the volume and pressure which they had when instantaneously formed, effect without work a change, the opposite of combustion, such that the powder is again completely formed; then allow the remaining compressed gas, which will be only the original air of the powder-chamber, to expand, pushing the projectile before it to the original seat of the projectile. Subtract this last work, done by the compressed air, from that required to push the projectile back from the muzzle

to the point where the powder is formed, and the result will be the work done in the case of instantaneous explosion with expansion to the muzzle.

Again, suppose as the projectile returns from the muzzle down the bore, that, by some means, we compress the parts of the powder-gas in succession and reform the powder, all the powder being formed by the time the projectile reaches its seat. To accomplish this, the opposite of progressive combustion and expansion, suppose the gases held in any way, as by suitable partitions which may be fixed or moveable as desired, so that the portions may be compressed unequally, and as required. Whenever any part arrives at its volume, pressure and temperature of formation, suppose it to be changed to its original solid state. Evidently the work done in the two cases will be exactly the same, as the same gas will be compressed through the same amount, and the question of simultaneous or successive compression will play no part.

55. Approximate Value of z.—We may write (10)

$$z = \frac{\tilde{\omega}\delta}{.0361\omega\Delta^2} \cdot \frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right); \quad . \quad . \quad . \quad (116)$$

$\frac{\Delta}{\delta}$ is in practice approximately $\frac{1}{2}$ (see Paragraph 5). It varies, roughly speaking, from about $\frac{4}{9}$ to $\frac{5}{8}$. Substituting these values, $\frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right)$ varies from $\frac{20}{81}$ to $\frac{15}{64}$, or is approximately $\frac{1}{4}$, this being its exact value when $\frac{\Delta}{\delta} = \frac{1}{2}$; when it is a maximum. We will, therefore, write for z in the conditions of practice, that is, where the density of loading nearly equals the gravimetric density of the powder, the chamber being nearly full,

$$\frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right) = \frac{1}{4}; \quad . \quad . \quad . \quad . \quad . \quad (117)$$

whence, substituting in (116),

$$z = \frac{\tilde{\omega}\delta}{.1445\omega\Delta^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

56. Monomial Formula in Terms of Travel.—The binomial $1 - y^{-.4}$ may be written $1 - \left(\frac{u}{u+z}\right)^{.4}$. To convert (107) into a monomial formula involving u, we may substitute for V the

product of a power of $\frac{u}{z}$ and a constant, substituting in turn for z its value from (118).

Assuming that 4 and 9 are fair working values of $\frac{u}{z}$, in practice, the binomial $1 - \left(\frac{u}{u+z}\right)^{.4}$ is found to vary (by calculation between these limits) as the .29 power of $\frac{u}{z}$. Practice indicates the $\frac{1}{4}$ power as a good and sufficient approximation.

In (107), then, K being a constant, we will place

$$V = K\left(\frac{u}{z}\right)^{\frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (119)$$

substitute for z, its value from (118), and for ω, its value $\frac{\pi c^2}{4}$. Combining all constants in one, which we will denote by A^2, we have

$$V^2 = A^2 \frac{f\tilde{\omega}^{\frac{3}{4}}}{w} c^{\frac{1}{2}}\Delta^{\frac{1}{2}}u^{\frac{1}{4}};$$

or, extracting the square root,

$$V = A\left(\frac{f}{w}\right)^{\frac{1}{2}} \tilde{\omega}^{\frac{3}{8}}(c)\Delta^{\frac{1}{4}}u^{\frac{1}{8}}. \quad . \quad . \quad . \quad . \quad . \qquad (120)$$

When the powder used is the same, or has the same force in the various cases, f also is constant. For these cases, we write

$$V = A_1 \frac{\tilde{\omega}^{\frac{3}{8}}(\Delta c)^{\frac{1}{4}}u^{\frac{1}{8}}}{w^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (121)$$

If we fire a given quick powder in a gun, we may find A_1 from the observed velocity and noted elements of loading and firing. This value of A_1, if substituted in (121), will give us the numerical muzzle velocity formula for that particular quick powder in any gun. It is to be observed, however, in using the formula, that a powder may be quick for one gun and slow for another, and that the formula will not hold when the powder becomes slow.

EXAMPLES.

1. Using Schaghticoke navy rifle powder in the 3″ B. L. R. and 60-pdr. B. L. R., the elements are (pounds and feet):

3″ B. L. R.	60-pdr. B. L. R.
$\tilde{\omega} = .75$	$\tilde{\omega} = 6.$
$\Delta = .76$	$\Delta = .5392$
$u = 3.22$	$u = 7.67$
$w = 7.$	$w = 46.5$
$c = .25$	$c = .44$
$V = 983$	

Required V, the muzzle velocity, for the 60-pdr., assuming the powder quick in both cases.

By (108), $V = 1030$ f. s. (approx.). By (121), $V = 1100$ f. s. (approx.).

Measured, 1071 f. s.

(This is an extreme case, as in the 60-pdr., $\frac{\Delta}{\delta}$ varies considerably from $\frac{1}{2}$, δ being about 1.75; and the powder may not be quick for the 3″ B. L. R.)

2. Prove that between $y = 5$ and $y = 10$, $1 - y^{-.4}$ varies as $\left(\frac{u}{z}\right)^{.29}$.

Assume $$1 - y^{-.4} = K\left(\frac{u}{z}\right)^{\theta};$$

then $$\theta = \frac{\log(1 - 10^{-.4}) - \log(1 - 5^{-.4})}{\log 9 - \log 4}.$$

3. Prove that, in a gun using a quick powder, if the projectile is always pushed home to the same point, the muzzle velocity is proportional to the $\frac{5}{8}$ power of the weight of charge.

Δ varies as $\tilde{\omega}$ in (121).

4. Prove also, if the projectile is pushed home to different points in different loadings, that the muzzle velocity varies, very approximately, inversely as the $\frac{1}{4}$ power of the reduced length of the chamber.

Neglecting the change in the travel u, V varies as $\Delta^{\frac{1}{4}}$; the mark on the rammer will show, by inspection, the change in the reduced length of the chamber.

5. Using 122 lbs. of powder (German cocoa) in the navy 8″ B. L. R., the muzzle velocity is 1999 f. s. What would it be, if $\tilde{\omega}$ were 125 lbs.? The above $\frac{5}{8}$ rule (see Example 3) holds for slow powders as well as quick; the $\frac{6}{10}$ power also gives good results.

Ans. Following $\frac{5}{8}$ power, $V = 2029$ f. s.
$\frac{6}{10}$ power, $V = 2028$ f. s.

6. Using 7 kilograms of smokeless powder (BN_1-p) in a 15 cm. Canet gun of 45 calibres, projectile weighing 40 kilograms, the muzzle velocity is 583 metres per second, and with 9.75 kilograms of the powder, 743 m. s.; according to what power of the charge does the velocity vary in this case? *Ans.* .73.

7. In the Navy 6″ B. L. R. (Dolphin's gun), when the charge of German cocoa powder is 58 lbs. and the projectile 100 lbs., the muzzle velocity is 2000 f. s., and when 50 lbs., 1835 f. s.; according to what power of the charge does the M. V. vary?

Ans. .58.

8. In the bore of the 57 mm. Hotchkiss R. F. gun, using a charge of .46 kilograms of smokeless powder, BN_1, density 1.57, density of loading .519, and a projectile of 2.72 kilograms, the observed velocity when $u = 8.8$ decimetres is 543 metres per second, when $u = 20.2$ d. m., $V = 648.3$ m. s. and when $u = 28.45$ d. m., $V = 682.1$ m. s.; according to what power of u does V vary? *Ans.* $\frac{3}{16}$ (approx.).

9. Using medium black and brown powders, the same power of u, $\frac{3}{16}$, seems to hold approximately. Assuming the muzzle velocity of the 8″ B. L. R. 2030 f. s., the travel being 16.41 feet, what would it be if the travel were lengthened 5 calibres, or $3\frac{1}{3}$ feet?

Ans. M. V. = 2103 f. s.

10. In the Gâvre formula, an exponential function of y involving three constants is substituted for Y in (107). As y_0, the expansion of the chamber volume in any particular loading, bears a fixed ratio to y, the expansion of the initial air-space, the function assumed is generally of the form $K\,10^{-by_0^{-k}}$. In terms of y_0,

then, for any particular loading in a particular gun, $V = N 10^{-b y_0^{-k}}$. Using BN_1 powder (smokeless) in the 57 mm. H. R. F. G., as in Example 8, the net chamber volume being .887 (d. m.)^3, and calculating N and b (by the method of least squares) from the following observed velocities (assuming $k = \frac{5}{4}$), the numerical formula is

$$V = 747.64 \times 10^{-.6728 y_0^{-\frac{5}{4}}}.$$

Compare the observed and computed velocities at the various travels.

Travel, Decimetres.	Velocities, Metres. Observed.	Computed.
28.45	682.1	679.0
20.20	648.3	651.0
17.92	636.5	639.0
15.64	622.3	623.9
13.93	612.6	610.0
12.22	595.0	593.1
10.51	574.4	572.1
8.80	543.1	545.5

11. Assuming V a series of ascending powers of u with unknown coefficients, determine the coefficients from the above observed velocities, using four terms of the series.

Assume $V = A + Bu + Cu^2 + Du^3$. Substituting for u and V, we will have eight equations for the determination of four unknown quantities, A, B, C and D. Add the eight equations together; the result is the normal equation for A. Multiply each equation by the coefficient of B and add all together. The result is the normal equation for B. Proceed similarly with C and D, and from the four normal equations determine A, B, C and D.

Ans. $V = 354.097 + 30.385\,u - 1.072\,u^2 + .0144\,u^3$.

(If eight terms had been used, the final equation would satisfy all the observed results exactly.)

CHAPTER VI.

LAWS OF COMBUSTION OF GUNPOWDER.

57. Combustion, Least Dimension.—If a homogeneous cylinder of powder be coated on its outer curved surface with some non-combustible material, and be ignited at one of its exposed ends, the flame will seem instantly to spread over that end.

The cylinder will then burn lengthwise, and if the pressure of the surrounding medium remains constant, equal lengths will be burned in equal times. If ignited at both ends, the total time of burning of the grain will be diminished by one-half. If, now, the non-combustible envelope be removed, the grain will appear, simultaneously with its ignition, to be inflamed over its entire surface.

It will then burn radially with the same uniform speed as it does lengthwise. If the density of the grain or the pressure of the surrounding medium be changed, the velocity of combustion (constant while they are constant) will be changed also.

If the diameter of the grain is less than its height, the grain will disappear at the instant when the diameter is completely burned; if, in place of the diameter, the height is the lesser, the latter will determine the time of burning of the grain.

The dimension that determines the time of burning of a grain of gunpowder is called the *least dimension*. The time of burning in seconds of a powder grain in free air under standard conditions will be denoted by τ, and the least dimension of the powder-grain by l_τ.

Since any dimension of a grain of powder burns from its two extremities, we will have for a grain of gunpowder in air, under standard conditions,

$$\text{Velocity of combustion} = V = \frac{l_\tau}{2\tau}. \quad . \quad . \quad . \quad (122)$$

58. Portion of Grain Burned.—The portion of a grain burned in air in any time t, not greater than τ, will be some function of $\frac{t}{\tau}$, and this function, $\varphi\left(\frac{t}{\tau}\right)$, vanishing when $t = 0$, may evidently be expressed by a series of ascending powers of $\frac{t}{\tau}$. The particular form of series used is

$$\varphi\left(\frac{t}{\tau}\right) = \frac{at}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2} + \ldots\right). \quad . \quad (123)$$

This may be rendered independent of time by writing in place of t and τ, the lengths burned in those times.

If, then, l is the sum of the lengths burned in air from the two ends of a dimension in time t, we will have for the portion of a grain burned in time t, or when l is burned,

$$\varphi\left(\frac{l}{l_\tau}\right) = a\frac{l}{l_\tau}\left(1 - \lambda\frac{l}{l_\tau} + \mu\frac{l^2}{l_\tau^2} + \ldots\right). \quad . \quad (124)$$

If we are given the portion of the least dimension that is burned, by placing $\frac{l}{l_\tau} = \gamma$, we will have the portion of grain burned; that is,

$$\varphi(\gamma) = a\gamma(1 - \lambda\gamma + \mu\gamma^2 +) \quad . \quad . \quad . \quad . \quad . \quad (125)$$

In case $t = \tau$, or $l = l_\tau$, or $\gamma = 1$, the whole grain is burned,

$$\varphi(\gamma) = 1,$$

and
$$a(1 - \lambda + \mu +) = 1 \quad . \quad . \quad . \quad . \quad . \quad (126)$$

The coefficients a, λ, μ, etc., vary with the shape of the grain, but their value must always be such as to satisfy equation (126).

59. Spherical Grains.—If γ is the portion of the diameter of a spherical grain burned, $1 - \gamma$ is the portion left. Spheres being proportional to the cubes of their diameters, we have for the portion of grain left, $(1-\gamma)^3$. The portion of grain burned, therefore, is

$$\varphi(\gamma) = 1 - (1 - \gamma)^3,$$

$$\therefore \varphi(\gamma) = 3\gamma\left(1 - \gamma + \tfrac{1}{3}\gamma^2\right) \quad . \quad . \quad . \quad . \quad . \quad (127)$$

This equation may be written in the form of (123) or (124), when desirable, by substituting for γ, $\frac{t}{\tau}$ or $\frac{l}{l_\tau}$.

The only condition necessary in the deduction of (127) is the similarity of the volume left with the original volume. The cube, the rectangular cylinder of equal altitude and diameter, and the numerous regular figures that will circumscribe a sphere fulfil equally well this condition.

For grains of these shapes, namely, spheres, cubes, etc., we find, by comparison of (127) with (125),

$$a = 3, \qquad \lambda = 1, \qquad \mu = \tfrac{1}{3}.$$

60. Rectangular Parallelopipeds.—Denote the ratio of the least dimension to the other two by x and z respectively. The portion of the least dimension that is burned being γ, the portion left will be $1 - \gamma$. The portion of the second dimension burned will be $x\gamma$, the portion left, $1 - x\gamma$; and similarly the portion left of the third dimension will be $1 - z\gamma$. The portion of the grain left will, therefore, be $(1 - \gamma)(1 - x\gamma)(1 - z\gamma)$, and the portion burned will be

$$\varphi(\gamma) = 1 - (1 - \gamma)(1 - x\gamma)(1 - z\gamma).$$

Developing the second member, arranging according to ascending powers of γ, we have

$$\varphi(\gamma) = (1 + x + z)\gamma\left[1 - \frac{x + z + xz}{1 + x + z}\gamma + \frac{xz}{1 + x + z}\gamma^2\right], \qquad (128)$$

where
$$\gamma = \frac{t}{\tau} = \frac{l}{l_\tau}.$$

By comparison of (128) with (125), we find, for grains of the form of a rectangular parallelopiped,

$$a = 1 + x + z, \qquad \lambda = \frac{x + z + xz}{1 + x + z}, \qquad \mu = \frac{xz}{1 + x + z}.$$

When $x = z$, or when the parallelopiped has a square base, and the altitude is the least dimension, we have

$$a = 1 + 2x, \qquad \lambda = \frac{2x + x^2}{1 + 2x}, \qquad \mu = \frac{x^2}{1 + 2x}.$$

These are the values of a, λ and μ for a flat grain with a square base. When $z = 1$, that is, when the parallelopiped has a square base whose side is the least dimension; in brief, for a long, pencil-shaped grain of square base or section,

$$a = (2 + x), \qquad \lambda = \frac{1 + 2x}{2 + x}, \qquad \mu = \frac{x}{2 + x}.$$

The above values are equally applicable to the rectangular cylinder having bases inscribed in those of the parallelopiped; they are also applicable to any regular rectangular prism circumscribed about the cylinder.

When $x = z = 1$, the grain is a cube, and the values of a, λ, and μ are the same as before found.

61. Pierced Cylinders.—In cylinders pierced with a cylindrical axial hole, the area of the base is the product of the thickness and the circumference of the circle midway between the outer circle and the axial hole. The volume of a burning pierced cylinder is, then, proportional to the product of its thickness and altitude, as the circle midway will remain constant. Suppose the thickness the least dimension, and let the ratio of the thickness to the altitude be denoted by x. y being the portion of the thickness burned, $(1 - y)$ is the portion left. The portion of the altitude burned is xy. The portion left is $(1 - xy)$. The portion of the grain left is therefore $(1 - y)(1 - xy)$, and the portion burned is

$$\varphi y = 1 - (1 - y)(1 - xy).$$

Expanding and arranging in series, we have

$$\varphi(y) = (1 + x)\, y \left[1 - \frac{x}{1 + x} y\right]. \quad . \quad . \quad . \quad (129)$$

The portion burned is exactly the same expression when the least dimension is the height, in place of the thickness, inasmuch as the thickness and the altitude enter the expressions for the different volumes or portions in exactly the same way.

The rectangular parallelopiped circumscribing a pierced cylinder and pierced at the axis by a parallelopiped circumscribing the axial hole of the cylinder, the corresponding sides of the inner and outer parallelopipeds being parallel, follows the same law, (129). This

law is also true for the grain formed by substituting any similar, regular, circumscribing prisms, with corresponding sides parallel, for the above parallelopipeds.

For grains in the form of pierced cylinders, then, and for other regular prismatic forms regularly pierced, be the least dimension either the thickness or the height, we have (comparing (129) with (125))

$$a=(1+x), \qquad \lambda=\frac{x}{1+x}, \qquad \mu=0.$$

These values will hold very approximately for regular hexagonal prisms pierced with *cylindrical* holes, and are the values generally used.

62. Weight of Charge Burned.—A charge of powder contains a number of grains which are supposed to burn alike. The portion of grain burned is, then, the portion of charge burned; so that, the weight of powder burned at any time t (not greater than τ) is

$$\tilde{\omega}\varphi\left(\frac{t}{\tau}\right)=\tilde{\omega}a\frac{t}{\tau}\left(1-\lambda\frac{t}{\tau}+\mu\frac{t^2}{\tau^2}+\right), \quad . \quad (130)$$

or,

$$\tilde{\omega}\varphi\left(\frac{l}{l_\tau}\right)=\tilde{\omega}a\frac{l}{l_\tau}\left(1-\lambda\frac{l}{l_\tau}+\mu\frac{l^2}{l_\tau^2}+\right), \quad . \quad (131)$$

or,

$$\tilde{\omega}\varphi(\gamma)=\tilde{\omega}a\gamma\,(1-\lambda\gamma+\mu\gamma^2+). \quad . \quad . \quad . \quad . \quad . \quad (132)$$

It appears, with the regular forms of powder grains in use, that, in all these series, the coefficients of the higher powers of γ are 0.

63. Velocity of Emission.—The velocity of emission of a powder grain is the portion of a grain burned in air in an indefinitely small time divided by that time. The portion of grain burned in air in time t (not greater than τ) is (see (123))

$$\varphi\left(\frac{t}{\tau}\right)=a\frac{t}{\tau}-a\lambda\frac{t^2}{\tau^2}+a\mu\frac{t^3}{\tau^3}+.$$

The portion burned in time dt (at the end of period t) is the differential of this expression with respect to t. If we denote the velocity of emission by η, we have, then,

$$\eta=\frac{d\varphi\left(\frac{t}{\tau}\right)}{dt}=a\frac{1}{\tau}-2a\lambda\frac{t}{\tau^2}+3a\mu\frac{t^2}{\tau^3}+,$$

or, $$\eta = \frac{a}{\tau}\left(1 - 2\lambda\frac{t}{\tau} + 3\mu\frac{t^2}{\tau^2} + \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

For all the regular forms of powder grains in use, as t increases η decreases. This is due to the negative term containing the coefficient λ. It is desirable, as combustion proceeds in guns, that the rate of evolution of powder gas should decrease as little as possible (an increase is desirable), in order that the pressure may be kept up till the projectile reaches the muzzle. The smaller, then, the value of λ, the better the shape.

Supposing that during combustion all grains retain their general shape, then: The sphere or cube is the worst among the regular forms; the flat parallelopiped is better than the long or pencil form; the quill or macaroni form is the exact equivalent of the large thin disc perforated at the centre, the length of grain in one case being the difference of the radii in the other.

Powder grains may be classified according to the number of decreasing dimensions. The sphere and parallelopiped have three diminishing dimensions. The pierced cylinder has only two; along with it may be classed the long, pencil-shaped grain (parallelopiped or cylinder), the length being affected very little, as a factor, in comparison with width and thickness, so that we need consider but two diminishing dimensions. The very long pierced cylinder has practically but one decreasing dimension, the small proportional decrease in length being neglected; the thin perforated disc of large difference between external and internal radii, the thin unperforated disc of large diameter, and the flat parallelopiped of large sides (linear) follow the same rule. In the case of three diminishing dimensions, terms involving a, λ, and μ occur in the expression for the portion of grain burned. Where, among the regular forms, there are but two decreasing dimensions, a and λ alone occur, and where there is but one, a only.

64. Number of Grains per Pound.—The number of grains to the pound is evidently the reciprocal of the weight in pounds of one grain. If v is the volume of a grain of powder in cubic inches its weight is (see (1))

$$w = .03613v\delta,$$

and the number of grains to the pound is

$$N = 27.68 \frac{1}{v\delta}.$$

For spherical grains of radius r,

$$v = \frac{4}{3}\pi r^3.$$

Substituting for π its value, we have

$$N = \frac{6.61}{r^3\delta}, \quad \ldots \ldots \quad (135)$$

where r is in inches.

For cubical grains of side h (in inches),

$$v = h^3,$$

$$\therefore \quad N = \frac{27.68}{h^3\delta}. \quad \ldots \ldots \quad (136)$$

For parallellopipeds of sides a, b and h (in inches), similarly, we have

$$N = \frac{27.68}{abh\delta}. \quad \ldots \ldots \quad (137)$$

For pierced cylinders of radii R and r, and height h (in inches),

$$v = \pi h(R^2 - r^2),$$

and
$$N = \frac{27.68}{\pi h(R^2 - r^2)\delta} = \frac{8.82}{h(R^2 - r^2)\delta};$$

or, placing the thickness $R - r = b$,

$$N = \frac{8.82}{hb(R + r)\delta}. \quad \ldots \ldots \quad (138)$$

For hexagonal pierced prisms in terms of the side a, height h, and radius of axial hole r (in inches), we have, similarly,

$$N = \frac{10.65}{h(a^2 - 1.209r^2)\delta}; \quad \ldots \ldots \quad (139)$$

or, in terms of the half distance R between opposite exterior sides,

$$N = \frac{7.99}{h(R^2 - .907r^2)\delta}. \quad \ldots \ldots \quad (140)$$

65. Combustion Under Variable Pressure.—Let MN (Fig. 4) represent the burning surface of a powder grain (smokeless or otherwise). From this, as shown in the figure, issues a stream of powder-gas normally. Denote the pressure of the powder-gas at the instant of formation by F, and the pressure of the gaseous surrounding medium, outside the issuing streams of gas by P. Also place p equal to the varying pressure in the stream, and ρ equal to the variable mass of unit volume, in it. Suppose the cross-section of the stream to be unity, and assume anywhere in it a lamina of thickness ds and at right angles to it. The measure of dp, the accelerating force of the lamina, will be (mass $\times$ acceleration)

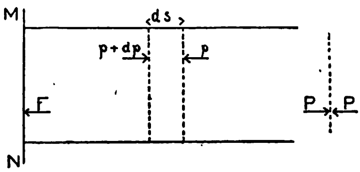


FIG. 4.

$$dp = \frac{dV}{dt}\rho ds = \frac{VdV}{ds}\rho ds = \rho VdV.$$

The transformation being adiabatic (see (36)),

$$p = k\rho^n \quad \text{and} \quad \rho = \frac{1}{k_1}p^{\frac{1}{n}};$$

$$\therefore \; VdV = \frac{k_1 dp}{p^{\frac{1}{n}}}.$$

The initial velocity of the issuing gas is 0 (approximately), and we will denote the final by V_1.

Integrating between proper limits, combining constants in one when possible, and remembering that the velocity increases as the pressure decreases,

$$\int_0^{V_1} VdV = k_1 \int_P^F p^{-\frac{1}{n}}dp, \text{ or } V_1^2 = k_2\left(F^{\frac{n-1}{n}} - P^{\frac{n-1}{n}}\right);$$

$$\therefore \; V_1 = k_2^{\frac{1}{2}}\left(F^{\frac{n-1}{n}} - P^{\frac{n-1}{n}}\right)^{\frac{1}{2}}.$$

This is the velocity with which the gas passes a point in the stream where the pressure is P. The mass of unit volume of the

gas at the same time will be determined by

$$\rho_1 = \frac{P^{\frac{1}{n}}}{k_1}.$$

and the mass of gas that flows past the above point in unit of time will be

$$\rho_1 V_1 = K_1 P^{\frac{1}{n}} \left(F^{\frac{n-1}{n}} - P^{\frac{n-1}{n}} \right)^{\frac{1}{2}}.$$

But if the density of the powder grain is constant, the mass of gas formed, which will be the same as that which passes any point, will be proportional to the velocity of combustion. We may, therefore, write for the velocity of combustion,

$$V' = KP^{\frac{1}{n}} \left(F^{\frac{n-1}{n}} - P^{\frac{n-1}{n}} \right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad (141)$$

If F could be increased, the velocity of combustion might be increased indefinitely. If P is constant, we know that V' is; therefore F is constant. The surface of the powder grain next the heated gas is evidently in a state of unstable equilibrium, held in check by the pressure, and if the pressure on the surface is too great, it will remain in that state. As the pressure is lessened by expansion of the gas, the reaction already begun on the surface is completed and gas is again produced, raising the pressure till again too great, stopping the reaction as before, and so on.

In order to keep the mass of gas flowing past a fixed point in the stream constant, the density of the powder being variable, the "velocity of combustion must vary inversely as the density." This law was first enunciated, as the result of experiment, by General Piobert.

If P is very small as compared with F, we may assume the quantity in brackets in (141) as constant (dropping the last term) and write

$$V' = k'P^{\frac{1}{n}},$$

or, substituting for n its value 1.4,

$$V' = k'P^{.7}.$$

This agrees with the experiments of M. de Saint Robert. He filled a lead tube with gunpowder, drawing the tube out and cut-

ting it into equal lengths. These lengths were burned at different altitudes above the sea. From these experiments, M. de Saint Robert concluded that the velocity of combustion of gunpowder varied as the $\frac{2}{3}$ power of the pressure of the surrounding medium. Captain Castan verified the increase of velocity of combustion with pressure by means of a tube containing powder with a hole for the escape of the gas. By changing the size of the hole, he changed the pressure.

As P begins to be appreciable comparatively with F, the quantity in brackets may be represented approximately by the product of a constant and a small negative power of P. This power increasing (negatively) is still quite small for the pressures employed in guns. Its effect is balanced by the use of a smaller power of P than .7.

As a sufficient approximation we will, in future, employ a principle which is undoubtedly the result of the closest observation, that of M. Sarrau: The velocity of combustion of gunpowder (in guns) varies as the square root of the pressure of the surrounding medium. That is,

$$V = kP^{\frac{1}{2}}. \qquad (142)$$

It may be noted, in the deduction of (141), that no account is taken of the non-gaseous residue of gunpowder. The close agreement of the formula with M. de Saint Robert's experiments however seems to indicate that the consideration of the residue would effect a needless complication of the subject. Moreover, this consideration would be impossible unless the residue were supposed formed in its entirety at the instant of combustion, and there is no real reason to suppose that it is not a result of expansion.

For smokeless powder, and for other explosives leaving no residue, in process of combustion as distinguished from detonation, the above equations should hold with exactness.

The combustion of powder in the atmosphere seems to be somewhat spasmodic. Powder grains, blown from guns, are often found partially burned, having ceased to burn in the atmosphere. Certainly, then, we may always expect local stoppages of combustion under such circumstances. The combustion may again be started by inflammation from adjoining portions of the grain,

and proceed as before; but, while comparable with similar cases, this alternation of combustion and inflammation is not comparable with combustion such as takes place in a chamber, in which the temperature of every part is higher than the temperature of ignition of the powder.

Note.—In the deduction of equation (141), the original velocity of the issuing gas is assumed 0. As the same mass of gas passes a point at pressure F as a point at pressure P, it is apparent that the original velocity of the gas is not 0, unless F is infinite. The original velocity in any case must be in the inverse ratio of the densities at F and P to the final velocity, each velocity being measured relatively to the retreating surface of the powder grain. The original velocity should then be $\left(\frac{P}{F}\right)^{\frac{1}{n}} V_1$.

On integration, we should then have

$$V_1^2\left(1-\left(\frac{P}{F}\right)^{\frac{2}{n}}\right)=k_2\left(F^{\frac{n-1}{n}}-P^{\frac{n-1}{n}}\right).$$

Neglecting the motion of the surface of the grain as a part of V_1, this will give us an expression of the same form as that in paragraph 65, when P is so small comparatively with F, that it may be neglected as an increase or decrease of F. The second member of (141) should in the more general case be divided by the square root of the quantity in parentheses in the first member of the above equation.

Examples.

1. Find the law of burning of a cylinder of powder of radius R, if it is perforated axially by a cylinder of radius r, and burns in this axial cylinder only.

$$\varphi(\gamma)=\frac{2r}{R+r}\gamma\left(1+\frac{R-r}{2r}\gamma\right).$$

2. Find the law of burning of a long, thin, pierced cylinder of smokeless powder. At the limit, making $x=0$, $\varphi(\gamma)=\gamma$.

3. Find the law of burning of a long, solid cylinder of smokeless powder.

At the limit, making $x=0$ in the values of a, λ, and μ for the pencil-shaped grain, $\varphi(\gamma)=2\gamma(1-\frac{1}{2}\gamma)$.

4. Show that the expression for $\varphi(\gamma)$ for the spherical grain of powder becomes a true maximum when $\gamma=1$.

$$\frac{d\varphi(\gamma)}{d\gamma}=0, \text{ when } \gamma=1.$$

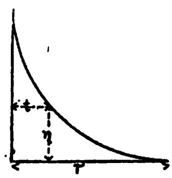
5. Show that the expression for $\varphi(\gamma)$ for pierced cylinders, where $x = \frac{1}{2}$, has a true maximum at an impossible value of γ.

$$\frac{d\varphi(\gamma)}{d\gamma} = 0, \text{ when } \gamma = 1\tfrac{1}{2}.$$

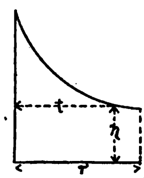
6. Show that the expression for $\varphi(\gamma)$, for the long, pencil-shaped grain, has a true maximum when $\gamma = 1$.

7. Trace the curves for the velocities of emission of the three general shapes of grain in use, in terms of the time of burning. (See (133)).

Sphere or Cube.



Rectangular Parallelopiped.



Pierced Cylinder.

Similar to the last figure, the curve becoming a straight line.

8. Find the number of grains of powder to the kilogram, given the volume of a grain in litres (cubic decimetres).

Ans. $N = \dfrac{1}{v\delta}$.

9. Find the number of spherical grains of $\frac{3}{4}''$ diameter to the pound; $\delta = 1.7$. *Ans.* $N = 74$ —.

10. Find the number of discs $2''$ in diameter and $\frac{1}{2}''$ high, per pound; $\delta = 1.867$. Also the values of a, λ and μ for this grain (see parallelopiped). *Ans.* $N = 10$ —.

$$a = \tfrac{3}{2},\ \lambda = \tfrac{3}{8},\ \mu = \tfrac{1}{24}.$$

11. How many pierced hexagonal prisms together weigh a pound, the distance between faces being $1\frac{1}{2}''$, radius of axial hole $\frac{1}{4}''$, and the height of grain $1''$, given $\delta = 1.818$? *Ans.* 8.7.

12. Assuming, in the case of the spherical grain, that the density of grain of gunpowder varies along the radius following the law $\rho = \left(\dfrac{r}{R}\right)^m \rho_1$, where ρ is the density at any point of radius r, and ρ_1 the density at the outside radius R. Find m.

$$\text{The mean density} = \frac{\int_0^R r^2 \rho\, dr}{\int_0^R r^2 dr} = \frac{3\rho_1}{m+3}.$$

If ρ_1 has the value which it would have with a perfectly incorporated solid mixture of the ingredients of black gunpowder in the ordinary proportions, $\rho_1 = 1.985$.

If the mean density δ (the given density of the grain) is 1.7, we will have

$$\delta = 1.7 = \frac{3 \times 1.985}{m + 3};$$

$$\therefore \quad m = .35.$$

13. If a cylinder varies radially, as in Example 12, prove that

$$\text{Mean density} = \frac{2\rho_1}{m + 2};$$

and if a pencil varies lengthwise alone, as in Example 12, or a pierced cylinder through its thickness (out and in), the half thickness taking the place of r and R above,

$$\text{Mean density} = \frac{\rho_1}{m + 1}.$$

14. The rate of evolution of *weight* of gas is given by $\eta = V\rho S$, where V is the velocity of combustion, and S the varying surface of the grain.

Hence, if the grain is a sphere, and $V = \frac{c}{\rho^a}$, and $\rho = \left(\frac{r}{R}\right)^m \rho_1$,

$$\eta = 4\pi \,.\, c \frac{R^{m(a-1)}}{\rho_1^{a-1}} \,.\, r^{2-m(a-1)}.$$

Hence, unless $m(a - 1) > 2$, η decreases as r decreases. Other things being equal, according to General Piobert, $a = 1$.

CHAPTER VII.

MUZZLE VELOCITY AND MAXIMUM PRESSURE FORMULÆ.

66. Velocity with Slow Powders.—The velocity of the projectile when slow powders are used in a gun is found, as before stated, from Equation (115), where $\tilde{\omega}\theta(y)$, the weight of powder burned to the point under consideration, is as yet unknown. It is not greater, though, than $\tilde{\omega}$. If γ is the portion of the least dimension burned to the point under consideration, $\tilde{\omega}\theta(y) = \tilde{\omega}\varphi(\gamma)$; and, substituting the value of $\tilde{\omega}\varphi(\gamma)$ from (132) in (115), we have

$$V^2 = Hi\frac{f}{w}Y\tilde{\omega}a\gamma(1 - \lambda\gamma + \mu\gamma^2 +). \quad . \quad . \quad . \qquad (143)$$

67. To Find γ.—Extracting the square root of each member of (115), and substituting for V its equivalent,

$$\frac{du}{dt} = \frac{zdy}{dt},$$

(see (103) to (104)), we have

$$\frac{zdy}{dt} = \left(Hif\frac{\tilde{\omega}}{w}Y\theta(y)\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \qquad (144)$$

In finding the velocity of combustion as a step in the calculation of γ, the pressure of the medium surrounding the powder is assumed to be that which the powder-gas would have if the powder were to cease burning, and all parts were to gain equilibrium. This is the pressure of the expanded powder-gas given, when the whole charge $\tilde{\omega}$ is burned, by (97). The pressure when a weight $\tilde{\omega}\theta(y)$ is burned is obtained by the substitution of $\tilde{\omega}\theta(y)$ for $\tilde{\omega}$. As this pressure of the surrounding medium corresponds exactly with p' in (110), we will denote it by p'. We have, then,

$$p' = \frac{f\tilde{\omega}\theta(y)}{.03611\,\omega z y^{1.4}}. \quad . \quad . \quad . \quad . \quad . \qquad (145)$$

The velocity of combustion at the standard atmospheric pressure, which we will denote by p_0, is (see (122)) $\frac{l_\tau}{2\tau}$. For a pressure p', it is, following (142), assuming that τ is the time occupied in combustion alone at the atmospheric pressure (without local stoppages or inflammation),

$$V=\frac{l_\tau}{2\tau}\left(\frac{p'}{p_0}\right)^{\frac{1}{2}};$$

$$\therefore \frac{V}{l_\tau}=\frac{1}{2\tau}\left(\frac{p'}{p_0}\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad (146)$$

Now,

$$V=\frac{dl}{2dt}=\frac{l_\tau d\gamma}{2dt};$$

hence, from (146),

$$\frac{d\gamma}{dt}=\frac{1}{\tau}\left(\frac{p'}{p_0}\right)^{\frac{1}{2}}.$$

Replacing p' by its value, from (145),

$$\frac{d\gamma}{dt}=\frac{1}{\tau}\left(\frac{f\bar{\omega}\,\theta(y)}{.0361\,\omega p_0 z y^{1.4}}\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad (147)$$

Dividing (147) by (144), member by member, to eliminate dt and and $\theta(y)$, multiplying through by zdy, replacing H and Y by their values (see (106) to (107)), and integrating between proper limits,

$$\gamma=\frac{1}{\tau}\left(\frac{zw}{5igp_0\omega}\right)^{\frac{1}{2}}\int_1^y\frac{dy}{[y(y^{.4}-1)]^{\frac{1}{2}}}. \quad . \quad . \quad (148)$$

The last integral is evidently a function of y alone. We will denote it by Y_1, whence

$$Y_1=\int_1^y\frac{dy}{[y(y^{.4}-1)]^{\frac{1}{2}}}. \quad . \quad . \quad . \quad . \quad (149)$$

Substitute for ω its value in terms of calibre, or

$$\omega=\frac{\pi c^2}{4},$$

and, for simplicity, place the constant coefficient

$$\left(\frac{4}{5gp_0\pi}\right)^{\frac{1}{2}}=k, \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

(148) then becomes

$$\gamma = \frac{k}{\tau c}\left(\frac{zw}{i}\right)^{\frac{1}{2}} Y_1 . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

68. General Velocity Formulæ.—The above values of γ, when substituted in formula (143), will give another velocity formula for all points along the bore of a gun.

In (143) place

$$Hif\frac{\tilde{\omega}}{w}a = x_0^2 , \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

and in (151) place

$$\frac{k}{\tau c}\left(\frac{zw}{i}\right)^{\frac{1}{2}} = x_1 , \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

whence

$$\gamma = x_1 Y_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (154)$$

Equation (143) becomes, on substituting values from (152) and (154), and placing

$$YY_1 = Y_0 , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

$$V^2 = x_0^2 x_1 Y_0 (1 - \lambda x_1 Y_1 + \mu (x_1 Y_1)^2 +) . \quad . \quad . \quad (156)$$

Extracting the square root of each member of (156), retaining two terms as sufficiently approximate, we have

$$V = x_0 x_1^{\frac{1}{2}} Y_0^{\frac{1}{2}}\left(1 - \frac{\lambda}{2} x_1 Y_1 + \right) ; \quad . \quad . \quad . \quad (157)$$

(156) and (157) are general forms that may be used when the various factors are known.

69. Muzzle Velocity Formulæ of M. Sarrau.—The approximate value of z, in ordinary cases of loading, is given by (118).

Replacing ω by $\frac{\pi c^2}{4}$, in (118), we have

$$z = \frac{\tilde{\omega}\delta}{.03 6 1 \pi c^2 \Delta^2} ; \quad . \quad . \quad . \quad . \quad . \quad (158)$$

$$\therefore \; z = 8.818 \frac{\tilde{\omega}\delta}{c^2 \Delta^2} , \quad . \quad . \quad . \quad . \quad . \quad (159)$$

an approximate value for z in inches, when $\tilde{\omega}$ is in pounds, and c in inches.

Substituting this value of z in (153), we have

$$x_1 = \frac{k}{\tau c^2 \Delta}\left(\frac{8.818 \tilde{\omega} \delta w}{i}\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \qquad (160)$$

The value of the function Y_1 (149) varies, for ordinary expansions (from $y = 1.1$ to $y = 10$, as will be seen later), approximately as $(y - 1)^{\frac{1}{2}}$. This approximate power agrees well with practice, making the muzzle velocity vary as $\tilde{\omega}^{\frac{5}{8}}$, which is very approximate, and, moreover, it greatly simplifies the formula for velocity.

We place, then, as approximate for all values of y,

$$Y_1 = k_1(y - 1)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (161)$$

where k_1 is a new constant. This may be written,

$$\text{since} \quad y = \frac{u + z}{z},$$

$$Y_1 = k_1\left(\frac{u}{z}\right)^{\frac{1}{2}},$$

and, remembering (159) and placing

$$\frac{k_1}{(8.818)^{\frac{1}{2}}} = k_2,$$

$$Y_1 = k_2 c \Delta \left(\frac{u}{\tilde{\omega}\delta}\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (162)$$

Substituting from (159) in (119) which is approximate for ordinary *muzzle distances* (not *all* values of y, but from 5 to 10 or more), we have

$$Y = \Delta k_3^{\frac{1}{2}} c^{\frac{1}{2}}\left(\frac{u}{\tilde{\omega}\delta}\right)^{\frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (163)$$

k_3 being another constant.

From (155), (162), and (163),

$$Y_0 = k_2 k_3 c^{\frac{3}{2}} \Delta^{\frac{3}{2}}\left(\frac{u}{\tilde{\omega}\delta}\right)^{\frac{3}{4}}. \quad . \quad . \quad . \quad . \quad . \qquad (164)$$

Substituting in (157) the values of x_0, x_1, Y_0 and Y_1, as given in (152), (160), (164) and (162) respectively, calling the small fractional powers of δ and i constants, omitting all but the first two terms inside the brackets, and combining all constant coefficients of the term outside the brackets in a new one, A, and all constant

coefficients of the second term inside the brackets in a new one, B, we have

$$V = A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}(\bar{\omega}u)^{\frac{1}{4}}\left(\frac{\Delta}{wc}\right)^{\frac{1}{2}}\left[1 - B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c}\right]. \quad (165)$$

This is M. SARRAU'S MUZZLE VELOCITY FORMULA.

70. Application to Use.—To use the above formula (165), the same kind of powder is fired in two dissimilar guns. The quantities in (165) that depend on the class of powder are f, a, τ, and λ. The powder being the same in the two cases, $\frac{fa}{\tau}$ and $\frac{\lambda}{\tau}$ are constants. We may then put

$$A\frac{fa}{\tau} = A',$$

and

$$B\frac{\lambda}{\tau} = B',$$

where A' and B' are constants.

The formula for velocity, using the same kind of powder in various guns, becomes

$$V = A'(\bar{\omega}u)^{\frac{1}{4}}\left(\frac{\Delta}{wc}\right)^{\frac{1}{2}}\left[1 - B'\frac{(wu)^{\frac{1}{2}}}{c}\right]. \quad . \quad . \quad (166)$$

The muzzle velocities being observed by chronograph in the two cases above referred to, we form two equations from (166), in which the only unknown quantities are the constants A' and B'. The values of A' and B' may then be determined, and these values will hold in any other gun using the same powder. The particular units used make no difference, but the units must be constant in all cases. The quantities generally used in this book, in the practical working of the *velocity* formula, are either abstract or in *feet* or *pounds*.

71. The Determination of A' and B'.—Dividing the two equations formed from (166), one by the other, member by member, we eliminate A'. From the resulting equation we determine B', and, by substitution in either of the two originals, we may determine A'. Or we may proceed as follows :

Place $$\frac{1}{K}=(\tilde{\omega}u)^{\frac{3}{8}}\left(\frac{\Delta}{wc}\right)^{\frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (167)$$

and $$H=\frac{(wu)^{\frac{1}{2}}}{c}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (168)$$

and let us indicate by subscripts the first or second firing. We have, then, from (166),

$$V_1=\frac{A'}{K_1}(1-B'H_1),$$

$$V_2=\frac{A'}{K_2}(1-B'H_2).$$

From these simultaneous equations we obtain,

$$A'=\frac{H_1K_2V_2-H_2K_1V_1}{H_1-H_2}, \quad . \quad . \quad . \quad . \quad . \qquad (169)$$

and $$B'=\frac{K_2V_2-K_1V_1}{H_1K_2V_2-H_2K_1V_1}. \quad . \quad . \quad . \quad . \quad . \qquad (170)$$

72. General Formulæ for Effective Pressures.—The effective accelerating pressure may be found by multiplying the acceleration of the projectile at any point by the mass of the projectile and dividing the result by the area of the base of the projectile. The acceleration (see (103)) is $\frac{d(V^2)}{2du}$. We may, then, write for the effective accelerating pressure at any point,

$$P=\frac{2wd(V^2)}{\pi gc^2du}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (171)$$

or, since $du=zdy$ (see (103) to (104)),

$$P=\frac{2wd(V^2)}{\pi gc^2zdy}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (172)$$

Differentiating (156), we have for P, then,

$$P=\frac{2wx_0^2x_1}{\pi gc^2z}\left(\frac{dY_0}{dy}-\lambda x_1\frac{d(Y_0Y_1)}{dy}+\mu x_1^2\frac{d(Y_0Y_1^2)}{dy}+\right); \qquad (173)$$

or, making $$\frac{dY_0}{dy}=X_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (174)$$

$$\frac{d(Y_0Y_1)}{dY_0}=X_1, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (175)$$

$$\frac{d(Y_0 Y_1^2)}{dY_0} = X_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (176)$$

and, placing X_0 outside the parenthesis,

$$P = \frac{2wx_0^2 x_1}{\pi g c^2 z} X_0 (1 - \lambda x_1 X_1 + \mu x_1^2 X_2 +); \quad . \quad . \qquad (177)$$

(171), (173), and (177) are general formulæ for the effective pressure at any point in the bore of a gun.

73. **Maximum Effective Pressure.**—The functions X_1 and X_2 become 0 when $u = 0$ or $y = 1$. Their coefficients are small, and as the maximum pressure is known to occur when the projectile has traveled but a short distance, it is probable that at the travel at which the maximum pressure occurs, the values of second and third terms of (177) may, as an approximation, be assumed zero. On these suppositions, the function X_0, though beginning at 0, must arrive at a maximum when y is still small. Whatever this maximum value of X_0 may be, it is a constant. Denoting it by k, and the maximum effective pressure by P_0, (177) becomes

$$P_0 = \frac{2wx_0^2 x_1}{\pi g c^2 z} k, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (178)$$

or, substituting for x_0^2 and x_1 their values from (152) and (153), calling i a constant, and joining all constants in one, k_1,

$$P_0 = k_1 \frac{fa}{\tau} \frac{\tilde{\omega} w^{\frac{1}{2}}}{c^3 z^{\frac{1}{2}}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (179)$$

Substituting the approximate value of z from (159) calling $\delta^{\frac{1}{2}}$ a constant, and combining all constants in a new one, K_0, we have

$$P_0 = K_0 \frac{fa}{\tau} \Delta \frac{(w\tilde{\omega})^{\frac{1}{2}}}{c^2}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (180)$$

which is M. Sarrau's Formula for the Maximum Effective Pressure on the projectile.

74. **Maximum Pressure.**—The pressure on the breech block is, as before mentioned, greater than that on the projectile. Their ratio evidently becomes greater as $\tilde{\omega}$ is increased, and smaller as

w is increased, being nearly constant (whether the powder is all burned or not) when $\dfrac{\tilde{\omega}}{w}$ is constant. The ratio may then be expressed very approximately by the product of some power of $\dfrac{\tilde{\omega}}{w}$ and a constant. Practice indicates that the maximum pressure varies as the $\frac{7}{4}$ power of the weight of the charge very approximately, while, remembering that Δ varies as $\tilde{\omega}$, formula (180) indicates that maximum effective pressure varies as the $\frac{3}{2}$ power. The power of $\dfrac{\tilde{\omega}}{w}$ by which to multiply the effective pressure is then approximately $\frac{1}{4}$, and we may write, combining all constants in a new one, K, and denoting the maximum pressure as shown by a pressure gauge in the face of the breech block by P_M,

$$P_M = K\frac{fa}{\tau}\Delta\frac{w^{\frac{1}{4}}\tilde{\omega}^{\frac{3}{4}}}{c^2}, \quad . \quad . \quad . \quad . \quad . \quad (181)$$

which is M. Sarrau's Formula for Maximum Pressure.

75. Use of Pressure Formula.—A gun is loaded in the ordinary way, a pressure gauge being placed in the face of the breech block, and the gun is fired. The maximum pressure, indicated by the gauge, and the values of Δ, w, $\tilde{\omega}$, and c are substituted in formula (181).

From this, the value of $K\dfrac{fa}{\tau}$ is calculated, it being

$$K\frac{fa}{\tau} = P_M\frac{c^2}{\Delta w^{\frac{1}{4}}\tilde{\omega}^{\frac{3}{4}}}. \quad . \quad . \quad . \quad . \quad . \quad (182)$$

This calculated value of $K\dfrac{fa}{\tau}$ will then answer for the same powder in any other gun.

We may evidently write the maximum pressure formula for any particular powder,

$$P_M = K'\Delta\frac{w^{\frac{1}{4}}\tilde{\omega}^{\frac{3}{4}}}{c^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (183)$$

where $$K' = K\frac{fa}{\tau}.$$

EXAMPLES.

1. The following are the actual data in the case of German cocoa powder (C_{82}) when fired in the two Navy 6″ B. L. R. known as the South Boston gun and Dolphin's gun. All data are given in feet and pounds.

	South Boston Gun.	Dolphin's Gun.
Weight of charge,	$\tilde{\omega} = 29.125$	$\tilde{\omega} = 50$
Travel to muzzle,	$u = 10$	$u = 11.62$
Density of loading,	$\Delta = .8763$	$\Delta = .9886$
Weight of projectile,	$w = 51$	$w = 100$
Calibre,	$c = .50$	$c = .50$
Muzzle velocity,	$V = 1685$	$V = 1836$

It is required to find the values of A' and B' in (166).

Ans. $A' = 497.6$.
$B' = .0014004$.

2. Derive from the values of A' and B' in Example 1, for the cocoa powder there defined, the formula giving the velocity at any point near the muzzle of the 8″ B. L. R. when loaded as follows, the units being feet and pounds:

$$\tilde{\omega} = 125 \qquad \Delta = .9049$$
$$w = 250 \qquad c = .6667.$$

Ans. $V = 825.85u^{\frac{3}{8}} - 27.428u^{\frac{7}{8}}$.

3. The travel to the muzzle in the last example is 16.41 feet. What is the muzzle velocity? *Ans.* M. V. = 2041 f. s.

4. From the result in Example 2, deduce an expression for the accelerating pressure, per square inch, in the 8″ B. L. R. at points near the muzzle. Use (171), taking c in inches.

Ans. $P = V\dfrac{619390 - 48000u^{\frac{1}{2}}}{12948u^{\frac{5}{8}}}$.

5. From Examples 3 and 4, show that the effective pressure in the 8″ B. L. R. as the shot clears the muzzle is

$P = 11654$ pounds per sq. in.

6. The maximum pressure in the 8″ B. L. R., using 122 lbs. of C_{82} powder, is 15.2 tons. What would it be if 125 lbs. were used?

The gun being the same, write (183), $P_M = k\tilde{\omega}^{\frac{7}{4}}$.

Ans. $P_M = 35500$ lbs.

7. It is required to determine A' and B′ by means of the records of the two firings with Dupont's black sphero-hexagonal powder of $\delta = 1.78$ given below. The data are, in feet and pounds:

6″ B. L. R. (S. B.).	60-pdr. B. L. R.
$\tilde{\omega} = 32$	$\tilde{\omega} = 10$
$\Delta = .9627$	$\Delta = .8987$
$u = 13.08$	$u = 7.67$
$w = 75.4$	$w = 46.5$
$c = .50$	$c = .44$
$V = 2001$	$V = 1427$

$A' = 810.50.$

$B' = .00570.$

8. It is required to find the muzzle velocity in the 6″ B. L. R. (South Boston) when loaded with the powder defined in Example 7, as stated below in *pounds, feet* and *seconds.*

$\tilde{\omega} = 23$	$u = 13.08$
$\Delta = .692$	$c = .50$
$w = 67$	$V = 1748$ (measured).

By calculation, we find $V = 1742.$

9. The recorded maximum pressure determined from the mean of 10 rounds in the 6″ B. L. R., Mark II. (which has a chamber slightly larger than the Dolphin's gun), using 53¼ lbs. of C_{82} powder, was 13.9 tons. What should the pressure be, using 50 lbs. in the same gun? *Ans.* 27900 lbs.

10. It is required to compare calculated with measured pressures in the following record of firings in the 6″ B. L. R. (Dolphin's gun, chamber capacity = 1400 cubic inches), with black sphero-hexagonal powder of 60 granules to the pound, $\delta = 1.817$, and $w = 100$ pounds, all the projectiles being identical in every respect.

$\tilde{\omega} = 35$ lbs.	$P_M = 16900$ bs. per square inch.	$V = 1506$ f. s.
" 40 "	" 22350 "	" 1657 "
" 45 "	" 26650 "	" 1760 "
" 47 "	" 27850 "	" 1792 "
" 49 "	" 30700 "	" 1880 "

For convenience, the formula may be written

$$P = k\tilde{\omega}^{\frac{7}{4}};$$

whence, from the first record, we find $k = 33.55$. The other calculated pressures are therefore 21350, 26240, 28310, 30450.

11. It is required to find A' and B' (166) for the use of Dupont's brown prismatic O. P. powder, $\delta = 1.818$, in any gun. The following data were given by two firings:

South Boston Gun.	8″ B. L. R.
$\tilde{\omega} = 25$ pounds.	$\tilde{\omega} = 110$ pounds.
$w = 68$ pounds.	$w = 250$ pounds.
$u = 12.5$ feet.	$u = 16.41$ feet.
$C = 920$ cu. in.	$C = 3824$ cu. in.
$c = .5$ feet.	$c = .6667$ feet.
$V = 1716$ f. s.	$V = 1949$ f. s.
$P_M = 7.9$ tons.	$P_M = 15.6$ tons.

Ans. $A' = 625.00$.
$B' = .002998$.

12. It is required to find A' and B' for Dupont's brown prismatic O. Q. powder, $\delta = 1.821$, in any gun. The following data were given by two firings:

8″ B. L. R., Mark I.	6″ B. L. R., Mark II.
$\tilde{\omega} = 110$ pounds.	$\tilde{\omega} = 49$ pounds.
$w = 250$ pounds.	$w = 100$ pounds.
$u = 16.41$ feet.	$u = 12.21$ feet.
$\Delta = .83085$	$\Delta = .95061$
$c = .66667$ feet.	$c = .50$ feet.
$V = 1959$ f. s.	$V = 1987$ f. s.
$P_M = 16.0$ tons.	$P_M = 14.0$ tons.

Ans. $A' = 602.5$
$B' = .00275$.

13. Using this powder in the 5″ B. L. R., with the following data,

$\tilde{\omega} = 30$ lbs. $\quad u = 10.25 \quad c = .41667$ feet,

$w = 60$ bs. $\quad \Delta = .93790$

what is the muzzle velocity? *Ans.* M. V. = 1895 f. s.

14. Prove that Equation (179) may be written for use with the same powder

$$P_0 = k_2 \left(\frac{w\tilde{\omega}\delta\Delta}{\delta - \Delta} \right)^{\frac{1}{2}} \frac{1}{c^2}.$$

15. Show that if in addition, the gun, weight of projectile, and size of chamber be fixed, (179) may be written in the form

$$P_0 = k_3 \left(\frac{\delta\Delta^2}{\delta - \Delta} \right)^{\frac{1}{2}}.$$

CHAPTER VIII.

CHARACTERISTICS, CHANGES IN ELEMENTS, MAXIMUM POWDERS.

76. **Powder Characteristics.**—In Paragraph (70), for any particular powder, we placed in the general muzzle velocity formula of M. Sarrau

$$A' = A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (184)$$

and

$$B' = B\frac{\lambda}{\tau}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

These values of A' and B' are fixed for each powder, and might be tabulated for the various known powders. It is usual, however, to proceed differently.

The quantities A and B are independent of the powder used, and of the elements of firing, depending alone upon the units. The quantities A' and B' are therefore proportional to $\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}$ and $\frac{\lambda}{\tau}$ respectively. The latter are called the *characteristics* of the powder, and are denoted by α and β respectively; that is,

$$\alpha = \left(\frac{fa}{\tau}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

$$\beta = \frac{\lambda}{\tau}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

f and τ are not known with exactness for any powder, but we may for comparison select some particular powder as a standard, and for this standard assume $f = 1$, and $\tau = 1$.

For the standard powder, then,

$$\alpha = \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} = a^{\frac{1}{2}},$$

and

$$\beta = \frac{\lambda}{\tau} = \lambda.$$

From the values of A' and B' for standard powder, we obtain the values of A and B for all powders, and we tabulate for the various powders, the values of α and β when known. Writing the muzzle velocity formula,

$$V = A\alpha(\tilde{\omega}u)^{\frac{3}{8}}\left(\frac{\Delta}{wc}\right)^{\frac{1}{4}}\left[1 - B\beta\frac{(wu)^{\frac{1}{2}}}{c}\right], \quad . \quad . \quad (188)$$

in which are given A and B, we have only to substitute the value of α and β to find the formula for any particular powder. Or, given the final formula, we may find α and β.

77. Another Standard.—It is apparent that the characteristics themselves may be assumed unity for the standard powder; the values of A and B will then be those of A' and B' for the chosen standard. Tabulating the characteristics on this basis would furnish a method of comparison equally efficacious with the preceding one.

78. Maximum Pressure.—The formula for the maximum pressure in terms of the characteristics is (see (181) and (186))

$$P_M = K\alpha^2\Delta\frac{w^{\frac{1}{2}}\tilde{\omega}^{\frac{3}{4}}}{c^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (189)$$

From this it is apparent that the maximum pressure is proportional to the quantity α^2, which may be found from it as readily as from the velocity formula. If, then, we determine K from the maximum pressure when using the standard powder, we may determine the maximum pressure when using another powder by consulting a table of characteristics.

A problem may be susceptible of solution in certain cases in another way; as, for example, when we are given the characteristics of one powder, and the maximum pressures in a gun for that powder and a second, and it is required to find the characteristics of the second, the composition of the powder remaining the same.

79. Shape of Grain.—The quantities a and λ depend on the shape of the powder grain. Their values for the various grains in use are given in Chapter VI. If only the shape of the grain is changed, we may readily find the ratio of the new characteristics to the old and substitute in (188) and (189), finding the new velocity and pressure formulæ.

80. Least Dimension.—The composition of the powder remaining constant, any change in the least dimension makes a proportional change in τ. If the shape and least dimension both change, but not the composition, we may readily find the new velocity formula, given the old. For, in the formula (165) f is constant, a, λ, and τ have suffered known changes, and all the remaining factors remain constant. Or, we may note the changes in the characteristics, tabulating them if desirable, and using (188).

81. Powder Ingredients.—Changing the powder ingredients will generally change f and τ, as τ depends not alone on the least dimension, but on the velocity of combustion as well (see (122)). Given the old and the new velocity formula, the ratio of the new values of f and τ to the old may be found; the ratio of the new to the old τ being found from the second term of the velocity formula, or from β; after which that of the new to the old f may be found from the first, or from α. The change in f may be found also by the use of the maximum pressure formula, which gives quite reliable results.

82. Summary.—If the shape of the grain is changed, a and λ change in the muzzle velocity formula, and a in the maximum pressure formula. If the least dimension is changed, τ is changed proportionally to it in the muzzle velocity and maximum pressure formulæ. If the ingredients change, f and τ will in general both change.

83. Weight of Charge.—It is apparent when the weight of charge changes that, in formulæ (165) and (188), $\tilde{\omega}$ and Δ both change. As Δ is proportional to $\tilde{\omega}$, V varies as the $(\frac{3}{8}+\frac{1}{4})$ or $\frac{5}{8}$ power of the weight of charge; whence, if everything else remains the same, we may write, A_0 being a constant,

$$V = A_0 \tilde{\omega}^{\frac{5}{8}}. \qquad (190)$$

This is true for breech-loading guns and for muzzle-loading guns in which the chamber remains constant.

In muzzle-loading guns, where the projectile is pushed home hard against the charge, so that the density of loading is practically the gravimetric density of the powder (a constant), V varies as the $\frac{3}{8}$ power of the weight of charge. There is, however, another

change in this case, due to variation in travel of the projectile. In the breech-loading gun the maximum pressure varies as the $\frac{7}{4}$ power of the weight of the charge; but, in a muzzle-loading gun, if the density of loading is constant, as the $\frac{3}{4}$ power. (See (181).)

84. Projectile not Home.—In breech-loading guns, when the projectile is not pushed home as indicated by the mark on the rammer handle, the chamber is decreased, and the length of travel increased. z, the reduced length of the initial air-space, is diminished by the length that the mark falls short.

According to the approximate expression (159), Δ is inversely proportional to the square root of z, if $\tilde{\omega}$ remains constant.

Δ may be found approximately as above indicated, or it may be found exactly by the reduction in chamber volume. Δ is inversely proportional to the volume of the chamber. Given the reduced length of the chamber, u_0, and the distance the rammer mark is short, take their difference. Δ varies inversely as the ratio of the result to the reduced length of the chamber. The *muzzle velocity* varies as the $\frac{1}{4}$ power of Δ nearly, and the *maximum pressure* as Δ.

The travel of the projectile, u, must be increased by the same quantity that z or u_0 is decreased. In guns using quick powders (see (121)), the muzzle velocity varies as the $\frac{1}{8}$ power of the travel of the shot, or

$$V = A_0 u^{\frac{1}{8}} \Delta^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

Returning to slow powders, we see that the second term of (180) becomes very small for very large values of τ; in this case, the muzzle velocity would vary approximately as the $\frac{3}{8}$ power of the travel of the projectile. The slow powders in use keep between limits $\frac{1}{8}$ and $\frac{3}{8}$, and well removed from the latter. For the slowest powders in use, the best power of u to use in the velocity formula appears to be $\frac{1}{4}$; whence, for them, we may write

$$V = A_2 u^{\frac{1}{4}} \Delta^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

For many intermediate powders, M. Sarrau recommends the power $\frac{3}{16}$; that is,

$$V = A_3 u^{\frac{3}{16}} \Delta^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (193)$$

Wherever the projectile may be on loading, the maximum pressure varies as Δ, or (see (181) or (189)),

$$P_M = K_1 \Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (194)$$

In formulas (191), (192), (193) and (194), the quantities A_0, A_2, A_3 and K_1 are constants which represent the unchanged elements of firing.

85. Monomial Muzzle Velocity Formula.—To obtain a formula of the type of (192) or (193) from (165), it is sufficient to assume the quantity inside the brackets proportional to a power of u. To obtain (192), the required power is evidently $u^{-\frac{1}{8}}$. The quantity in brackets in (165) is evidently the same power of the whole second term, that it is of of $u^{\frac{1}{2}}$; the power of u being $(-\frac{1}{8})$, that of $u^{\frac{1}{2}}$ is $(-\frac{1}{4})$, and we may therefore write, K_2 being a new constant,

$$1 - B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = K_2\left(\frac{\tau c}{B\lambda(wu)^{\frac{1}{2}}}\right)^{\frac{1}{4}},$$

or,

$$1 - B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = \frac{K_2}{B^{\frac{1}{4}}}\left(\frac{\tau c}{\lambda}\right)^{\frac{1}{4}}\left(\frac{1}{wu}\right)^{\frac{1}{8}}. \quad . \quad . \qquad (195)$$

Substituting this in (165), we have, combining all constants in a new one, M,

$$V = M\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{1}{4}}\left(\frac{\tilde{\omega}}{w}\right)^{\frac{3}{8}}(\Delta u)^{\frac{1}{4}}. \quad . \quad . \qquad (196)$$

Proceeding in the same way with (193), which also should be derived from (165), M_1 being a new constant,

$$V = M_1\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}}\frac{\tilde{\omega}^{\frac{3}{8}}\Delta^{\frac{1}{4}}c^{\frac{1}{8}}u^{\frac{3}{16}}}{w^{\frac{7}{16}}}. \quad . \quad . \quad . \qquad (197)$$

These monomial formulæ are due to M. Sarrau. The first, (196), is for very slow powders. It presents the curious feature of being independent of the calibre. The second is for quicker powders, though not so quick as to be completely burned in the gun. The binomial formula (165) will not hold for these last.

86. Similar Guns, Similarly Loaded.—Two guns are similar when all their homologous linear dimensions are proportional to the calibre.

They are *similarly loaded* when the weight of charge and projectile are proportional to the cube of the calibre, and the grains of powder are alike in form and composition, with dimensions proportional to the calibre. Returning to the general formula for velocity using slow powders, or (156), we see that Y_0 and Y_1, being functions of y alone, are constant when y is constant. Therefore, when y or $\frac{u}{z}$ is constant, as in similar guns, we may write

$$V^2 = A_4 x_0^2 x_1, \quad \ldots \ldots \ldots \quad (198)$$

A_4 being a new constant. When the guns are similarly loaded, we see that in the value of x_0^2 (see equation (152) $\bar{\omega}$ varies as w, and that everything else is constant. Hence, x_0^2 is constant. Likewise. in similar guns, similarly loaded, x_1 is constant. For, in (153) τc varies as $(zw)^{\frac{1}{3}}$. Hence, with slow powders, in similar guns similarly loaded, the velocities of the projectiles are the same for the same values of $\frac{u}{z}$.

Again, referring back to (126), we see that the limiting value of

$$a\gamma(1 - \lambda\gamma + \mu\gamma^2 +)$$

is

$$a(1 - \lambda + \mu +) = 1. \quad \ldots \ldots \ldots \quad (199)$$

The limiting value of $\bar{\omega}\theta(y)$, the powder burned at expansion y, therefore, is

$$\bar{\omega}a(1 - \lambda + \mu +) = \bar{\omega}. \quad \ldots \ldots \ldots \quad (200)$$

(156) becomes for the limiting case, remembering (154) and (155),

$$V^2 = \frac{x_0^2}{a} Y, \quad \ldots \ldots \ldots \quad (201)$$

which, of course, is only a new way of writing the formula for velocity in a gun, at travel u, when all the powder is burned before the projectile reaches it (see (107)).

x_1 is constant in similar guns similarly loaded, as before shown; Y_1, being a function of y alone, is constant for the same value of y, and therefore of $\frac{u}{z}$. $\gamma = x_1 Y_1$ becomes unity for the same value of $\frac{u}{z}$ in two similar guns similarly loaded; the powder is therefore completely burned in the two guns at the same value of $\frac{u}{z}$. Inside of this point, the formula for velocity using slow powders applies; outside, that for quick powders. x_0^2 is constant for the quick powder as it was for the slow, and V is constant for similar guns similarly loaded with quick powders, at the same value of $\frac{u}{z}$.

In the effective pressure formula (177), the different functions of y, namely Y_2, Y_3, and Y_4, being constant for the same value of y, and therefore of $\frac{u}{z}$,

x_0^2 and x being constant, as in the case of velocities, for the same value of $\frac{u}{z}$, and w being proportional to c^2z in similar guns similarly loaded, P is constant.

We may, therefore, enunciate the following principle, due to M. Sarrau :—
"*In similar guns similarly loaded, the velocities and pressures corresponding to distances passed over proportional to the calibre are equal.*"

87. **Maximum Powders.**—We have supposed the quantity i, which represents the ratio of the mechanical energy of the projectile to the total work of the system, to be a constant for different powders ; i probably is not exactly a constant for any two powders. It seems that, all other conditions of loading remaining the same, there is a size of grain that gives in a gun a maximum of velocity. Powders composed of grains of this size are called *maximum powders*. The maximum powder changes with every condition of loading. It must of course be a quick powder, and that it is not the instantaneous powder seems to be definitely settled. The more nearly instantaneous the powder is, the higher the maximum pressure, the more the gun is strained, and probably, speaking in general terms, the greater the quantity of waste work. The loss of heat from the powder gas, which has heretofore been assumed zero, will affect the velocity, and it would seem, though not by any means certain, that the instantaneous powders and those nearly so, would again be at a disadvantage. It may not be far wrong, then, to assume the maximum powder as the slowest quick powder, or the quickest slow powder ; in other words, the powder that is just burned in the gun. For this powder,

$$\tilde{\omega}\theta(y) = \tilde{\omega}a(1 - \lambda + \mu +) = \tilde{\omega}\,,$$

$$a(1-\lambda + \mu +) = 1\,,$$

and
$$(1-\lambda +) = \frac{1}{a}\,.$$

When, therefore, in formula (143),

$$1-\lambda\gamma + \mu\gamma^2 + = \frac{1}{a}$$

γ is at its limit 1. Continuing on to (156), when the quantity in parentheses equals $\frac{1}{a}$, γ is unity. In (157), similarly, when the quantity in parentheses equals $\frac{1}{a^{\frac{1}{2}}}$, we are using powder that is just completely burned under the given conditions. Continuing to (165), derived from (157), we may still write approximately for maximum powders,

$$1 - B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = \frac{1}{a^{\frac{1}{2}}}\,;$$

whence
$$B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = 1 - \frac{1}{a^{\frac{1}{2}}}\,, \quad \ldots\ldots\ldots \quad (202)$$

and
$$\tau = B\frac{\lambda(awu)^{\frac{1}{2}}}{c(a^{\frac{1}{2}}-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

Formula (165) should be quite accurate for spherical powders, inasmuch as the values of λ and μ, namely 1 and $\frac{1}{3}$, will allow a very approximate square root of the quantity in parentheses in (156) to be extracted.

For spherical powders $a = 3$, $a^{\frac{1}{2}} = 1.732$, and we have as a limiting value from (202), or for maximum spherical powders,

$$B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = .423. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

It may be remarked, inasmuch as, for spherical powders, $a\gamma(1-\lambda\gamma+\mu\gamma^2)$ has a true maximum when $\gamma = 1$, that the value of τ for the maximum powder may be obtained by placing the derivative, with respect to τ of the expression for muzzle velocity, equal to zero.

Formula (165) is not so accurate for pierced prismatic powders. For them, $\mu = 0$, and an approximate square root of the quantity in parentheses in (156) cannot be extracted.

For pierced prisms of a thickness equal to the half height, $a = \frac{3}{2}$, $a^{\frac{1}{2}} = 1.225$, and for maximum pierced prisms, we have, following (202),

$$B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{2}}}{c} = .184. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

Given the shape of the grain and the second term in parentheses in velocity formula (165), we can tell at once how near the grain is to a maximum. This second term must be less than $1-\frac{1}{a^{\frac{1}{2}}}$. The ratio of its value to $1-\frac{1}{a^{\frac{1}{2}}}$ will be the inverse ratio of the two times of burning or the inverse ratio of its least dimension to the least dimension of the maximum powder. The *derivative* of the velocity expression cannot be used for prismatic powders; the true maximum would occur when $\gamma > 1$.

Examples.

1. If in a certain gun 54 pounds of powder gave 2105 f. s. velocity with 15.6 tons pressure, what velocity and pressure will 52 pounds give with the same gun and powder?

Ans. $V = 2056$ f. s.
$P_M = 14.7$ tons.

2. Find the weight of charge necessary to obtain 1700 f. s. velocity in the 6-inch B. L. R. if 54 pounds of the same powder gave 2105 f. s. with 15.6 pressure.

Ans. $\tilde{\omega} = 38.36$ lbs.

3. Using Wetteren powder as a standard, (188) becomes (units, feet and lbs.)

$$V = 502.63a(\tilde{\omega}u)^{\frac{1}{2}}\left(\frac{\Delta}{wc}\right)^{\frac{1}{2}}\left[1 - .0058891\,\frac{(wu)^{\frac{1}{2}}}{c}\right].$$

Write the numerical formulæ for muzzle velocity with the following Navy powders.

Powder.	Description.	α	β	f	τ
Sp. Hex. 98....	Sp. Hexagonal, N = 98.......	1.2252	0.4214	1.1924	2.3753
Sp. Hex. 100..	" " N = 100......	1.2762	0.3497	1.2762	2.7944
German Cocoa,	Pierced Prismatic, N = 11½ ..	1.0922	0.2958	0.8950	1.1256
German Cocoa,	" " " " ..	1.0856	0.2527	1.0353	1.3177
LN Dupont....	Sp. Hexagonal, N = 100. ..	1.2755	0.6047	0.8968	1.6538
NU "	Pierced Prismatic, N = 11½ ..	0.9768	0.1804	1.1770	1.846
OP "	" " " " ..	1.0319	0.1848	1.2787	1.8014
OP "	" " " " ..	1.0800	0.3191	0.8114	1.0435
Standard......	Wetteren (13–16) Flat grain...	1.6036	0.851	1.0	1.0

4. With the same standard, but with c in inches,

$$P_M = 17820a^2\Delta\,\frac{\tilde{\omega}^{\frac{1}{2}}w^{\frac{1}{2}}}{c^2};$$

write the numerical formulæ for maximum pressures with the same powders as are given in Example 3.

5. What would be the maximum pressure in the 6″ B. L. R. (Dolphin's gun) using 35 lbs. of powder of the composition described in Example 10, Chapter VII., but consisting of spherical grains running 480 to the pound? (Treat sphero-hexagonal grains as spheres.) *Ans.* $P_M = 33{,}800$ lbs. τ is divided by 2.

6. The thickness of German cocoa (C_{82}) powder (from axial hole to outside) is about ½ inch; the maximum pressure, using it in the 8″ B. L. R. with all elements as in Example 2, Chapter VII., being 35,500 lbs., what would it be using spherical grains of ½-inch diameter? *Ans.* $P_M = 71{,}000$ lbs. a is doubled.

7. What would be the pressure using spherical grains of 1″ diameter? *Ans.* 35,500 lbs.

8. The composition of the powder (Example 7) is changed so that

the velocity of combustion in air is doubled, the force remaining the same, and the diameter of the grains is changed to $2\frac{1}{2}$ inches. What is the pressure? *Ans.* 28,400 lbs.

9. What would be the pressure using 50 lbs. of this last powder in the 6-inch B. L. R. (Dolphin's gun) of Example 10, Chap. VII.?

$$P_M = 28400 \times \frac{64}{.9048 \times (250)^{\frac{1}{4}} \times (125)^{\frac{3}{4}}} \times .9886 \times \frac{(100)^{\frac{1}{4}} \times (50)^{\frac{3}{4}}}{36}.$$

10. Referring back to Example 2, Chapter VII., we find, using German cocoa powder (C_{82}) as there specified, for points near the muzzle,

$$V = 825.85\, u^{\frac{3}{8}} - 27.428\, u^{\frac{7}{8}}.$$

Find a corresponding velocity formula, using the same elements, but changing the powder to that of Example 8.

Ans. $V = 738.7\, u^{\frac{3}{8}} - 32.914\, u^{\frac{7}{8}}$.

$$a_2 = 2a;\ \tau_2 = \tfrac{5}{2}\tau;\ \text{and } \lambda_2 = 3\lambda.$$

11. The German cocoa grain is supposed changed in shape and size to a spherical grain. The other elements in the 8″ B. L. R. and the maximum pressure remain the same. What is the muzzle velocity formula?

Ans. $V = 825.85\, u^{\frac{3}{8}} - 41.142\, u^{\frac{7}{8}}$.

$$P_M \text{ is constant, } \therefore \frac{a}{\tau} \text{ is constant; } \frac{\lambda}{\tau} \text{ changes.}$$

12. Compare the value of τ for the maximum spherical grain, obtained by placing the derivative of the velocity formula with respect to τ equal to 0, with that obtained by making $\gamma = 1$.

$$\left.\begin{aligned} &(1)\ B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{4}}}{c} = \frac{1}{3} \\ &(2)\ B\frac{\lambda}{\tau}\frac{(wu)^{\frac{1}{4}}}{c} = 1 - \frac{1}{\sqrt{3}} \end{aligned}\right\} \quad \therefore\ \tau_1 = (3 - \sqrt{3})\tau_2.$$

They should be the same; they differ because the term containing μ is in error.

13. It is required to find the correct granulation (maximum powder granulation) in the South Boston 6″ gun when loaded with

spherical powder, the composition and the other elements of loading being as given Example 7, Chapter VII.

$$\left.\begin{array}{r} .00570\dfrac{(wu)^{\frac{1}{3}}}{c} = .358 \\ 1 - \dfrac{1}{\sqrt{a}} = \dfrac{.732}{1.732} = .423 \end{array}\right\} \therefore\ \tau_0 = \frac{358}{423}\tau, \quad \text{and } R_0 = \frac{358}{423}R.$$

If the powder used consisted of 100 grains per pound, the maximum powder should be $\left(\dfrac{423}{358}\right)^3 \times 100 = 165$ per lb.

14. Prove that, in similar and similarly loaded guns, if the powder is maximum in one case, it is in the other also.

15. Writing the velocity formula, using pierced cylinders 1.5″ diameter, 1″ height and .5″ diameter axial hole,

$$V = A_3(1 - B_3 \times .333),$$

find the velocity formula, using discs 2″ diameter and .5″ height, of the same composition.

Ans. $V = A_3(1 - B_3 \times .375)$.

16. The velocity formula for the spherical grain 1″ in diameter of the same composition (see Example 15) is

$$V = A_3(1 - B_3 \times .500).$$

CHAPTER IX.

VELOCITY AND PRESSURE AT ANY POINT IN THE BORE OF A GUN.

88. Velocity Formula.—The formulæ of M. Sarrau for muzzle velocity and maximum pressure seem to be all that could be desired in their practical application. To find, however, the varying velocity and pressure along the bore of a gun requires more complex work. By differentiation of (165) or (166), the pressure at points near the muzzle may be found. The muzzle velocity formulæ will not themselves hold for points far inside the muzzle of an ordinary gun, being based on such a power of the expansions as is only approximately true for the variation in their number to the muzzles of guns as at present constructed. We must then return to formula (156).

Substituting in (156), the values of x_0 and x_1 from (152) and (153), calling i a constant, and combining all constant factors outside the brackets in a new one, M, and the constant factors of the second term in the brackets in a new one, N, we have

$$V^2 = M\frac{fa}{\tau}\cdot\frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}}Y_0\Big[1 - N\frac{\lambda}{\tau}\cdot\frac{(wz)^{\frac{1}{2}}}{c}Y_1$$
$$+ N^2\frac{\mu}{\tau^2}\frac{wz}{c^2}Y_1^2 + \Big]. \quad . \quad (206)$$

For the pierced cylindrical grain, $\mu + = 0$. We have therefore for grains having a similar value of μ,

$$\therefore \quad V^2 = M\frac{fa}{\tau}\cdot\frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}}Y_0\Big[1 - N\frac{\lambda}{\tau}\frac{(wz)^{\frac{1}{2}}}{c}Y_1\Big]. \quad (207)$$

For the sphere or cube, $\mu = \frac{1}{3}$. Assuming, which will cause small error, that for the spherical or cubical grain $\mu = \frac{1}{4}$, the quantity in brackets in (206) becomes a perfect square, and we may write

very approximately for the spherical or cubical grain, M_0 and N_0 being the new constants,

$$V = M_0 \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tilde{\omega}}{c}\right)^{\frac{1}{2}} \left(\frac{z}{w}\right)^{\frac{1}{4}} Y_0^{\frac{1}{2}} \left[1 - N_0 \frac{\lambda}{\tau} \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right]. \quad (208)$$

This formula is evidently very approximate for parallelopipeds that approach the cube in shape.

89. Determination of Constants.—Given a table of values of Y_1 and Y_0, (207) and (208) may be used in the same way as (165) when we know the results of the firings in two dissimilar guns.

$$M\frac{fa}{\tau} \text{ and } N\frac{\lambda}{\tau}, \text{ or } M_0\left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \text{ and } N_0\frac{\lambda}{\tau},$$

will be found in exactly the same way as are found

$$A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \text{ and } B\frac{\lambda}{\tau}.$$

Placing, in (207),

$$\frac{Mfa}{\tau} = M', \text{ and } N\frac{\lambda}{\tau} = N'$$

we have, for any one powder,

$$V^2 = M' \frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0 \left(1 - N' \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right). \quad (209)$$

Making

$$\left(\frac{\tilde{\omega}}{c}\right)\left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0 = \frac{1}{K} \text{ and } \frac{(wz)^{\frac{1}{2}}}{c} Y_1 = H,$$

denoting the particular firings by subscripts, we find,

$$M' = \frac{H_1 K_2 V_2^2 - H_2 K_1 V_1^2}{H_1 - H_2}, \quad (210)$$

and

$$N' = \frac{K_2 V_2^2 - K_1 V_1^2}{H_1 K_2 V_2^2 - H_2 K_1 V_1^2}. \quad (211)$$

In (208), if we place

$$M_0\left(\frac{fa}{\tau}\right)^{\frac{1}{2}} = M_1, \text{ and } N_0\frac{\lambda}{\tau} = N_1,$$

we have

$$V = M_1\left(\frac{\tilde{\omega}}{c}\right)^{\frac{1}{2}}\left(\frac{z}{w}\right)^{\frac{1}{4}} Y_0^{\frac{1}{2}}\left(1 - N_1 \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right). \quad (212)$$

If we make

$$\left(\frac{\tilde{\omega}}{c}\right)^{\frac{1}{2}}\left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0^{\frac{1}{2}} = \frac{1}{K} \quad \text{and} \quad \frac{(wz)^{\frac{1}{2}}}{c} Y = H,$$

and show different values of H and K in the particular firings by subscripts, M_1 and N_1 may be found by (169) and (170), their values being those given for A' and B' respectively.

90. Pressures.—From (177), if we substitute for x_0^2 and x_1 their values from (152) and (153), we have

$$P = \frac{2}{\pi c^2} \cdot \frac{1}{gz} \cdot M \frac{fa}{\tau} \cdot \frac{\tilde{\omega}}{c} (wz)^{\frac{1}{2}} X_0 \Big[1 - N \frac{\lambda}{\tau} \cdot \frac{(wz)^{\frac{1}{2}}}{c} X_1$$

$$+ N^2 \frac{\mu}{\tau^2} \cdot \frac{wz}{c^2} X_2 + \Big]. \quad . \quad (213)$$

If the velocity is found, by (209),

$$P = \frac{2}{\pi c^2} \cdot \frac{w}{gz} . M' \frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}} X_0 \left[1 - N' \frac{(wz)^{\frac{1}{2}}}{c} X_1 \right]. \quad (214)$$

If the velocity is found, by (212), we have

$$P = \frac{2}{\pi c^2} \frac{w}{gz} M_1^2 \frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}} X_0 \left(1 - 2N_1 \frac{(wz)^{\frac{1}{2}}}{c} X_1 + N_1^2 \frac{wz}{c^2} X_2 \right). \quad (215)$$

In all these formulæ, the data are generally in feet and pounds, as in the velocity formula, except the first c^2, which may be taken in square feet or square inches according as the pressure is desired per square foot or square inch. The formulæ apply to the calculation of the effective accelerating pressure at any point in the bore of a gun, using black or brown powders.

If we write the velocity formula

$$V^2 = M_2 Y_0 [1 - N_2 Y_1 + N_3 Y_1^2 +], \quad . \quad . \quad . \quad (216)$$

we may evidently write the effective pressure formula

$$P = \frac{2}{\pi c^2} \frac{w}{gz} M_2 X_0 [1 - N_2 X_1 + N_3 X_2 +] . \quad . \quad . \quad (217)$$

91. Calculation of Y_1.—In order to use the velocity and pressure formulæ 207), (208), (214), and (215), it is necessary to calculate Y_0, Y_1, X_0, X_1, and

X_2. The only calculation that is at all difficult is that of Y_1; all the others are based on it. We have (see (149))

$$Y_1 = \int_1^v \frac{dy}{\left[y\,(y^{.4} - 1)\right]^{\frac{1}{2}}}.$$

Place
$$y^{.2} = \sec\varphi.$$

Then
$$Y_1 = 5 \int_0^\varphi (\sec\varphi)^{\frac{5}{2}} d\varphi \quad \ldots \ldots \ldots \quad (218)$$

Integrating by parts (see Johnson's Integral Calculus, page 81), and making the proper transformations,

$$\int_0^\varphi (\sec\varphi)^{\frac{5}{2}} d\varphi = \frac{2}{3}(\sec\varphi)^{\frac{1}{2}} \tan\varphi + \frac{1}{3}\int_0^\varphi (\sec\varphi)^{\frac{1}{2}} d\varphi \quad \ldots \quad (219)$$

Now,
$$\sec\varphi = \frac{1}{\cos\varphi},$$

and
$$\cos\varphi = 1 - 2\left(\sin\frac{\varphi}{2}\right)^2.$$

Place $z = \sin\dfrac{\varphi}{2}$; then $\dfrac{\varphi}{2} = \sin^{-1} z$, and $d\varphi = \dfrac{2dz}{(1 - z^2)^{\frac{1}{2}}}$,

$$\therefore \int_0^\varphi (\sec\varphi)^{\frac{1}{2}} d\varphi = \int_0^z \frac{2dz}{(1 - z^2)^{\frac{1}{2}}(1 - 2z^2)^{\frac{1}{2}}}.$$

Place $z = \sqrt{2}\,x$. Then

$$\int_0^\varphi (\sec\varphi)^{\frac{1}{2}} d\varphi = \sqrt{2}\int_0^x \frac{dx}{\left(1 - \frac{x^2}{2}\right)^{\frac{1}{2}}(1 - x^2)^{\frac{1}{2}}}.$$

Transform back again to the use of angles, by placing

$$x = \sin\theta.$$

Then
$$\theta = \sin^{-1} x,$$

and
$$d\theta = \frac{dx}{(1 - x^2)^{\frac{1}{2}}},$$

whence
$$\int_0^\varphi (\sec\varphi)^{\frac{1}{2}} d\varphi = \sqrt{2}\int_0^\theta \frac{d\theta}{(1 - \frac{1}{2}(\sin\theta)^2)^{\frac{1}{2}}} \quad \ldots \quad (220)$$

This last integral is of the general form of

$$\int_0^\theta \frac{d\theta}{(1 - k^2(\sin\theta)^2)^{\frac{1}{2}}},$$

where $k = \sin 45^\circ$.

It has been tabulated under the heading F(45°) in Table IX., Tome II., Fonctions Elliptiques, by Legendre, for every value of θ from 0 to 90°.

Substituting (220) in (219) and (218), we have

$$Y_1 = \frac{10}{3}(\sec\varphi)^{\frac{1}{2}}\tan\varphi + \frac{5\sqrt{2}}{3}\int_0^\theta \frac{d\theta}{(1-\frac{1}{2}(\sin\theta)^2)^{\frac{1}{2}}}, \quad . \quad (221)$$

in which

$$\varphi = \sec^{-1}(y^{.2}),$$

and

$$\theta = \sin^{-1}\left(\sqrt{2}\sin\frac{\varphi}{2}\right).$$

92. **Calculation of Y and Y_0.**—The value of Y is found by (106). The value of Y_0 is calculated from Y and Y_1 by (155).

93. **Calculation of X_0.**—Differentiating (155), we have

$$dY_0 = YdY_1 + Y_1dY. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

From (113),

$$dY = .4y^{-1.4}dy = Xdy;$$

and dY_1 is obtained by dropping the integral in (149); that is,

$$dY_1 = \frac{dy}{[y(y^{.4}-1)]^{\frac{1}{2}}} = Y^{-.5}y^{-.7}dy. \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

Substituting in (222) and then in (174),

$$X_0 = \frac{dY_0}{dy} = Y^{.5}y^{-.7} + .4\,Y_1y^{-1.4},$$

or,

$$X_0 = Y^{.5}y^{-.7} + Y_1X. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

94. **Calculation of X_1.**—By (175) we have, differentiating Y_0Y_1,

$$X_1 = Y_1 + \frac{Y_0dY_1}{dY_0}.$$

Now (see (174) and (223)),

$$\frac{dY_1}{dY_0} = \frac{dY_1}{X_0dy} = Y^{-.5}y^{-.7}X_0^{-1.0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

$$\therefore \; X_1 = Y_1 + Y_0Y^{-.5}X_0^{-1.0}y^{-.7}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

95. **Calculation of X_2.**—Differentiating $Y_0Y_1^2$ in (176), we have

$$X_2 = Y_1^2 + 2\,Y_0Y_1\frac{dY_1}{dY_0}.$$

Hence, by (225),

$$X_2 = Y_1^2 + 2\,Y^{-.5}Y_0Y_1X_0^{-1.0}y^{-.7}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

or, substituting $Y_0^{-.5}$ for $Y_1^{-.5}Y^{-.5}$ (see (155)), we have

$$X_2 = Y_1^2 + 2\,Y_0^{-.5}Y_1^{-1.5}X_0^{-1.0}y^{-.7}.$$

We find, also,

$$X_2 = \frac{d(Y_0 Y_1^2)}{dY_0} = \frac{d[Y_1(Y_0 Y_1)]}{dY_0} = Y_0 Y_1 \frac{dY_1}{dY_0} + Y_1 \frac{d(Y_0 Y_1)}{dY_0}.$$

Substituting for $\frac{dY_1}{dY_0}$ from (225), and for $\frac{d(Y_0 Y_1)}{dY_0}$ its value X_1, we have

$$X_2 = Y_0 Y_1 Y^{-.5} X_0^{-1.0} y^{-.7} + Y_1 X_1;$$

or,

$$X_2 = Y_1(Y_0 Y^{-.5} X_0^{-1.0} y^{-.7} + X_1). \quad . \quad . \quad . \quad . \quad . \quad (228)$$

All parts of X_2 have been calculated before.

Tables containing the logarithms of the values of these various functions will be found at the end of the book.

96. **Power of $\frac{u}{z}$ Corresponding to Y_1.**—By a reference to the tables, it will be found that for values of y between 1.1 and 10, or between values of $\frac{u}{z}$ of .1 and 9.0,

$$Y_1 \text{ varies as } (y-1)^{.426} \text{ or as } \left(\frac{u}{z}\right)^{.426}.$$

The power of $\frac{u}{z}$, namely $\left(\frac{u}{z}\right)^{.5}$, assumed in the second member of M. Sarrau's formula for muzzle velocity is then fairly accurate for the whole length of bore.

Between $y = 1.1$	and $y = 1.2$,	Y_1	varies as	$(y-1)^{.4854}$	
" $y = 1.1$	" $y = 5.0$,	Y_1	"	$(y-1)^{.4425}$	
" $y = 5.0$	" $y = 6.0$,	Y_1	"	$(y-1)^{.3679}$	
" $y = 6$	" $y = 7$,	Y_1	"	$(y-1)^{.36006}$	
" $y = 7$	" $y = 8$,	Y_1	"	$(y-1)^{.3535}$	
" $y = 8$	" $y = 9$,	Y_1	"	$(y-1)^{.3488}$	
" $y = 9$	" $y = 10$,	Y_1	"	$(y-1)^{.3443}$.	

These powers of $(y-1)$ offer a convenient method of interpolation between the limits indicated, using first and second differences of the exponents, and in the tables the values of log Y_1 for fractional values of y were calculated in this way.

97. **Pressure on Breech-Block.**—Assuming that the non-gaseous products of the combustion of the charge follow the same law of distribution in rear of the projectile as the gaseous, or, in other words, that the mass of each lamina, composed partly of gaseous and partly of non-gaseous matter, is the same as if all

products were gaseous, and also, supposing the projectile perfectly free to move and the gun held firmly without recoil, the ratio between the effective and breech pressures in a gun is given by (66) and (68). In practice, however, this ratio depends somewhat on the point of ignition of the charge. Ignited at the rear end, the first tendency is to virtually increase the weight of the projectile by including with it a portion of the charge. Ignited at the forward end, the first tendency is for part of the charge to remain at rest. The results of these first tendencies may possibly be felt as far forward in the gun as the point of travel at which the maximum pressure occurs; if so, the ratio between the maximum effective and the maximum pressure would be different in the two cases without necessarily affecting the muzzle velocity, which would depend on the final distribution of the products.

The resistance due to forcing increases alike the pressure on breech and projectile.

98. Travel for Maximum Pressure.—Referring to the table, it will be seen that X_0 in the first term of the pressure formula arrives at a maximum when $y = 1.6$, or when $\frac{u}{z} = .6$; that is, when the travel of the projectile is .6 the reduced length of the initial air-space. The second and remaining terms of the pressure formula are increasing, but are very small for small values of y; the second, however, which is negative, is much greater than the succeeding terms. It may then be said with present forms of grains of black and brown powders (speaking accurately) that the maximum pressure occurs in a gun when the projectile has traveled somewhat less than $.6z$. If we could make a powder for which the sign of the second term were positive, the maximum pressure would occur when the projectile had traveled a distance greater than $.6z$. As a sufficient approximation generally we may write for the travel of projectile at the instant of maximum pressure, for very slow powders,

$$u_M = .6z, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

the coefficient becoming smaller as the powder becomes quicker. For instantaneous powders, we would substitute 0 for .6.

99. Maximum Powders.—The treatment of M. Sarrau's binomial formula in finding the maximum powder may be applied to the velocity formulæ of the present chapter. In the formula for V^2, (206) or (207), if the quantity in parentheses equals $\frac{1}{a}$, we are using the maximum powder. If the quantity in brackets in (208) equals $\frac{1}{a^{\frac{1}{2}}}$, we are using the maximum powder also (very nearly). Equation (208) being theoretically approximate for spherical or cubical powders, and the expression for $\varphi(\gamma)$ for these powders having a true maximum when $\gamma = 1$, the value of τ which will make V in (208) a maximum (as shown by placing the derivative of V with respect to τ equal to 0) should be very nearly that of τ for the maximum powder.

From (206), we have for the maximum powder

$$1 - N\frac{\lambda}{\tau}\frac{(wz)^{\frac{1}{2}}}{c}Y_1 + = \frac{1}{a},$$

or,

$$\tau = N\frac{a\lambda}{c} \cdot \frac{(wz)^{\frac{1}{2}}}{a - 1}Y_1 - . \quad . \quad . \quad . \quad . \quad . \qquad (230)$$

For pierced cylinders, we have

$$N\frac{\lambda}{\tau}\frac{(wz)^{\frac{1}{2}}}{c}Y_1 = \lambda. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (231)$$

From (208), or for spheres, etc., we have similarly,

$$\tau = N_0\frac{\lambda}{c}\frac{(awz)^{\frac{1}{2}}}{a^{\frac{1}{2}} - 1}Y_1. \quad . \quad . \quad . \quad . \quad . \qquad (232)$$

100. Weight of Powder Burned.—It is evident that the quantity in brackets in (206) or (207) will always indicate what portion of the powder is burned at any point in a gun. The quantity in brackets is in fact simply $(1 - \lambda\gamma + \mu\gamma^2 +)$. The given value of this, along with the known values of λ and μ, will allow us to find γ, and this, substituted in $a\gamma(1 - \lambda\gamma + \mu\gamma^2 +)$, where a is known, will give us at once the portion of each grain burned, and therefore the portion of the charge. For pierced cylinders, the second term in brackets of the formula for V^2 will give us $\lambda\gamma$ at once.

Neglecting in any case the value of μ, and writing the velocity formula (see (216))

$$V^2 = M_2 Y_0 (1 - N_2 Y_1), \quad . \quad . \quad . \quad . \quad . \quad (233)$$

the portion of the powder grain burned is (making $N_2 Y_1 = \lambda\gamma$),

$$\varphi(\gamma) = a\gamma(1 - \lambda\gamma) = \frac{a}{\lambda} N_2 Y_1 (1 - N_2 Y_1), \quad . \quad . \quad (234)$$

and the weight of powder burned,

$$\tilde{\omega}\varphi(\gamma) = \frac{\tilde{\omega} a}{\lambda} N_2 Y_1 (1 - N_2 Y_1). \quad . \quad . \quad . \quad . \quad (235)$$

Examples.

1. The following are the data in the case of German cocoa powder (C_{82}) when fired in the two Navy 6″ B. L. R. known as the South Boston gun and the Dolphin's gun. All data are given in feet and pounds:

	South Boston Gun.	Dolphin's Gun.
Weight of charge,	$\tilde{\omega} = 29.125$	$\tilde{\omega} = 50$
Travel of projectile to muzzle,	$u = 10.$	$u = 11.62$
Density of loading,	$\Delta = .8763$	$\Delta = .9886$
Weight of projectile,	$w = 51$	$w = 100$
Calibre,	$c = .5$	$c = .5$
Muzzle velocity,	$V = 1685$	$V = 1836.$

It is required to find the formula for velocity.

In the South Boston gun, according to (10) (in which it must be remembered the units are inches and pounds), $z = 1.4388$ feet; and in the Dolphin's gun, similarly, we have $z = 1.9413$ feet.

In the South Boston gun, at the muzzle,

$$y = \frac{u}{z} + 1 = \frac{10}{1.4388} + 1 = 7.9504.$$

In the Dolphin's gun, at the muzzle, $y = \dfrac{11.62}{1.9413} + 1 = 6.9856.$

By reference to Table I., it is found by interpolation that when $y = 7.9504$, or for the South Boston gun at the muzzle,

$$\log Y_0 = .54188 \text{ and } \log Y_1 = .79077.$$

Similarly, for the Dolphin's gun at the muzzle,

$$\log Y_0 = .50043 \text{ and } \log Y_1 = .76766.$$

Substituting for V, $\tilde{\omega}$, c, etc. their values in (209), using the notation $[\lg^{-1}]$ to express "the number of which the logarithm is" we have for the S. B. gun,

$$(1685)^2 = M'[\lg^{-1} 1.53239](1 - N'[\lg^{-1} 2.02458]),$$

and for the Dolphin's gun,

$$(1836)^2 = M'[\lg^{-1} 1.64448](1 - N'[\lg^{-1} 2.21274]).$$

From these two simultaneous equations we find (see (210) and (211)),

$$M' = [\lg^{-1} 4.98252] = 96055,$$

and $$N' = [\lg^{-1} 7.09757 - 10] = .0012519.$$

Consequently, for C_{82} powder in any gun we may write

$$V^2 = [\lg^{-1} 4.98252] \frac{\tilde{\omega}}{c}\left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0\left(1 - [\lg^{-1} 7.09757 - 10] \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right).$$

2. Verify the equation found in Example 1 in the 8″ B. L. R., using 125 pounds of German cocoa powder (C_{82}). The following are the data in feet and pounds:

$$\tilde{\omega} = 125, \qquad w = 250,$$
$$u = 16.41, \qquad c = .66667.$$
$$\Delta = .9048, \qquad V = ?$$

Proceeding as before, $z = 3.2673$,
$y = 6.0225$.

From Table I, $Y_0 = [\lg^{-1} .44942]$ and $Y_1 = [\lg^{-1} .74023]$. Substituting these values in the equation found in Example 1, we have for the velocity at the muzzle where $u = 16.41$ feet,

$$V = 2022 \text{ f. s.}$$

3. Using (212), find M_1 and N_1, and the muzzle velocity in the 8″ B. L. R.,

$$1685 = M_1[\lg^{-1} .76620](1 - N_1[\lg^{-1} 2.02458]),$$
$$1836 = M_1[\lg^{-1} .82224](1 - N_1[\lg^{-1} 2.21274]);$$

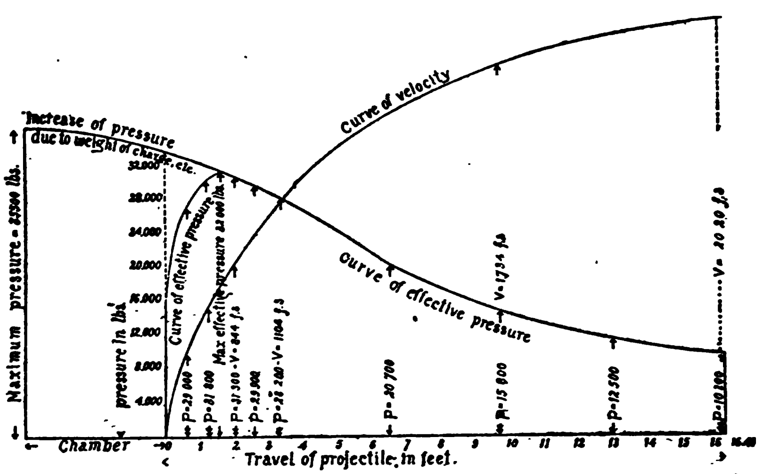
whence
$$M_1 = [\lg^{-1} 2.49305],$$
$$N_1 = [\lg^{-1} 6.83519 - 10],$$
$$V = 2028 \text{ f. s.}$$

The measured velocities for 3 rounds in the 8″ B. L. R., using 122 lbs. of German cocoa powder, were 1996, 1996, and 2006 f. s.; or, assuming $V = k\tilde{\omega}^{\frac{1}{3}}$, for 125 lbs., the velocities should be 2026, 2026, and 2036 f. s.

4. Find the effective pressure formulæ for all three guns from the results in Examples 1 and 2. (Use (214)).

In the S. B. gun,
$$P = [\lg^{-1} 8.29930 - 10][\lg^{-1} 5.97303]X_0(1 - [\lg^{-1} 8.33132]X_1).$$
In the Dolphin's gun,
$$P = [\lg^{-1} 8.45162 - 10][\lg^{-1} 6.12657]X_0(1 - [\lg^{-1} 8.54259]X_1).$$
In the 8″ gun,
$$P = [\lg^{-1} 8.37359 - 10][\lg^{-1} 6.31365]X_0(1 - [\lg^{-1} 8.72967]X_1).$$

5. Calculate a table of effective pressures for all three guns at various points, and construct velocity and pressure curves for the 8″ B. L. R., the maximum breech pressure being about 35,500 pounds.



	South Boston 6-in. B. L. R.		6-in. B. L. R. (Dolphin).		8-in. B. L. R.	
y.	u (feet).	P (lbs.).	u (feet).	P (lbs.).	u (feet).	P (lbs.).
1.1	.144	9583	..	..	..	..
1.2	.29	11900	.39	23435	0.65	28991
1.4	.58	13429	.78	26142	1.3	31777
1.6	.86	13546	1.16	26130	1.96	31319
1.8	1.15	13194	..	..	2.61	29897
2.0.	1.44	12675	..	..	3.27	28205
3.0	2.88	10021	..	..	6.53	20683
4.0	4.32	8131	5.82	14701	9.80	15768
5.0	5.76	6821	..	..	13.06	12506
6.0	7.19	5873	..	..	16.33	10217
7.0	8.63	5159	11.62	8847		
8.0	10.00	4598				

[The maximum pressures for the Dolphin's gun and the 8″ B. L. R., as compared with measured pressures, are nearly exact; that for the South Boston gun is about 1000 lbs. too high.]

6. In the Dolphin's gun, loaded as indicated in Example 1, what would be the least dimension of a grain of maximum powder of the shape and material of C_{82} powder?

$$\left.\begin{array}{l}\text{In brackets, second term of } V^2 = .2043 \\ \text{By form of grain,* } 1 - \dfrac{1}{a} = \lambda = .3333\end{array}\right\} \therefore \quad \tau_M = \frac{2043}{3333}\tau.$$

Taking the least dimension of C_{82} as .5 inches, the least dimension or thickness of the maximum powder grain is .307 inches.

7. What would be the least dimension of the maximum grain for the South Boston gun? *Ans.* .199 inches.

8. What portion of the charge is burned in the Dolphin's gun, using C_{82} powder, as in Example 1?

$$\lambda\gamma = \tfrac{1}{3}\gamma = .2043 \quad \therefore \quad \gamma = .6129.$$
$$\varphi\gamma = a\gamma(1 - \lambda\gamma) = \tfrac{3}{2}(.6129)(.7957) = .7315.$$
Ans. $\tilde{\omega}\varphi(\gamma) = 36.58$ lbs.

9. What weight of powder is burned in the S. B. gun, using C_{82} powder, as in Example 1? $\gamma = .3975$, $\varphi(\gamma) = .51725$.
Ans. $\tilde{\omega}\varphi(\gamma) = 15.065$ lbs.

10. The following is the data for brown cocoa C_2 in the 57 mm. Hotchkiss R. F. G.:

$c = .57$ dm. $\tilde{\omega} = .92$ kgms. $w = 2.72$ kgms. $z = 1.4$ dm.

* The German cocoa grain is a hexagonal prism pierced axially by a cylindrical hole, the thickness of the grain from the axial hole to the outside being one-half the height.

When $u = 8.8$ dm., $V = 488.7$ m. s. When $u = 20.20$ dm., $V = 600$ m. s. Find formulæ for velocity and pressure at various points in the bore. $V^2 = 87533\ Y_0\{1 - 0.027648\ Y_1\}$.

$$P = 3399.5 X_0\{1 - 0.027648 X_1\}.$$

11. Compute a table of velocities and pressures in the bore of a 57-mm. H. R. F. G., using brown C_2 powder:

BROWN COCOA (C_2) POWDER.*

$c = 0.57$ dm. $\tilde{\omega} = 0.92$ kg. $w = 2.72$ kg. $z = 1.400$ dm.

$y = \frac{u}{z} + 1$.	u decimetres.	u calibres.	P kg. per cm².	V metre-seconds.	$\tilde{\omega}(\varphi)\gamma$ kilograms.	$\varphi(\gamma)$.
1.0	0.0	0.0	0	0.0	0.0	0.0
1.1	0.14	0.246	1726	56.1	.075	.081
1.2	0.28	0.491	2135	90.5	.103	.112
1.3	0.42	0.737	2317	118.1	.124	.135
1.4	0.56	0.982	2395	141.6	.141	.153
1.5	0.70	1.228	2417	162.0	.155	.169
1.6	0.84	1.474	2407	180.2	.168	.182
1.7	0.98	1.719	2378	196.5	.179	.194
1.8	1.12	1.965	2337	211.4	.189	.205
1.9	1.26	2.211	2289	225.1	.198	.215
2.0	1.40	2.456	2238	237.7	.207	.225
2.1	1.54	2.702	2185	249.3	.215	.233
2.2	1.68	2.947	2132	260.2	.222	.241
2.3	1.82	3.193	2079	270.5	.229	.249
2.4	1.96	3.439	2027	280.1	.235	.256
2.5	2.10	3.684	1976	289.1	.241	.262
3	2.80	4.912	1747	327.9	.268	.291
4	4.20	7.369	1404	384.4	.308	.334
5	5.60	9.825	1168	425.1	.337	.367
6	7.00	12.281	998	456.6	.361	.393
7	8.40	14.737	871	482.2	.381	.414
8	9.80	17.193	772	503.6	.398	.433
9	11.20	19.650	692	522.0	.413	.449
10	12.60	22.106	627	537.9	.426	.463
11	14.00	24.562	513	552.1	.438	.476
12	15.40	27.018	527	564.7	.449	.488
13	16.80	29.474	488	576.2	.459	.498
14	18.20	31.930	454	586.6	.468	.508
15	19.60	34.386	424	596.1	.476	.517
16	21.00	36.842	397	605.0	.484	.526
21.321	28.45	50.000	297	642.6	.518	.563

* The results were computed by Capt. James M. Ingalls, 1st Artillery, U. S. A.

CHAPTER X.

MISCELLANEOUS.

101. Velocity Formula of Messrs. Noble and Abel.—Messrs. Noble and Abel assume that powder-gases in expansion absorb heat from the residue, the two remaining in equilibrium of temperature. This is the method of expansion already discussed in Paragraph 23. Beginning with formulæ (43), n' being given by (42), in which c_0 and c' are specific heats of the powder-gas under constant volume and constant pressure respectively, c_1 the specific heat of the residue, and β the ratio of its weight to that of the gas, and using (43) as we used (35), we find a formula for velocity at once by the substitution of n' for n in (105). We have, then,

$$V^2 = \frac{if\bar{\omega}g}{.0181w(n'-1)}(1 - y^{1-n'}). \quad . \quad . \quad . \quad . \quad . \quad (236)$$

Messrs. Noble and Abel change the form of this slightly. First, they practically assume the density of loading unity, which is approximate in guns ordinarily. Then, supposing the powder all burned before the projectile begins to move, $\omega z = C(1 - \alpha)$, in which α is the ratio defined in Paragraph 36. y of course represents the expansion of this volume, z being not exactly the reduced length of the initial air-space, but the reduced length of the initial space occupied by the powder-gases (which, in this case, where α is different from $\frac{1}{\delta}$, is a little changed). The density of loading being unity, the chamber pressure (see (87)) is

$$p_0 = \frac{f}{1-\alpha};$$

$$\therefore \quad f = p_0(1-\alpha).$$

Under the same supposition, following (2),

$$\bar{\omega} = .03613\ C.$$

Remembering (see (42)) that $n' - 1 = \dfrac{c' - c_0}{c_0 + \beta c_1}$,

we may write (substituting in (236))

$$V^2 = 2ig\frac{p_0 C(1-\alpha)(c_0 + \beta c_1)}{w(c' - c_0)}\left(1 - y^{\frac{c_0 - c'}{c_0 + \beta c_1}}\right) \quad . \quad . \quad . \quad . \quad (237)$$

The factor i is supposed constant for each gun and is employed as a factor of effect; that is, to make the theoretical coincide with the observed velocity of the projectile. Messrs. Noble and Abel do not suppose powder instanta-

neous, but tacitly assume that the muzzle velocity will be the same if the charge is burned in the gun as if it were burned before the projectile began to move. On account of its simplicity, tables being constructed for values of $i = 1$, formula (237) along with (43) is still much used. These formulæ do not, however, answer for slow powders, which are not completely burned in a gun, except in so far that i may be varied to fit any case.

In (237), following Messrs. Noble and Abel, p_0 is the tension of the permanent gases when $\Delta = 1$, or 6554 atmospheres.

c' the specific heat of the permanent gases at constant pressure, or .2324.

c_0 " " " " " " " " volume, or .1762.

β the ratio of the weight of non-gaseous to gaseous products, or 1.2957.

α the ratio of the volume of non-gaseous products to that of the chamber, or .57.

c_1 the specific heat of the non-gaseous product, or .45.

On substitution of the above values, n' becomes 1.074 and as the above value of p_0 is 43 tons per square inch, we may write, from (43), in tons per square inch, assuming the charge completely burned,

$$p = 43y^{-1.074};$$

or, denoting the volume of bore in rear of projectile at any point by v,

$$p = 43\left(\frac{.43C}{v - .57C}\right)^{1.074} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

Note.—If α were defined as the ratio of the volume of the residue to the volume of the chamber in which the charge is placed, the density of loading would necessarily be assumed constant in all cases, but it need not be 1. This latter is approximate, however, and in using it the values for α and f accord with those previously given.

102. **M. Helie's Formulæ.**—M. Helie's formula for velocity is an empirical one involving three constants, one of which is determined practically by firing and varies for each gun and condition of loading. The second and third constants seem to answer for all guns when using black powder and probably very approximately for brown powder. They also answer approximately for smokeless powders, but better results may be obtained by changing them. The formula is also known as the Gavre formula. It is

$$V = N 10^{-.6y_0^{-\frac{1}{2}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (239)$$

where y_0 is the expansion of the chamber volume. Given the muzzle velocity and expansion to the muzzle we find N. The formula will then give good results in the same gun with the same elements, at other points in the bore along the chase. It cannot be carried back to the point of maximum pressure.

Formula (239) (changing to the Napierian base and remembering that $\log_e 10 = 2.3026$) may be written

$$V = Ne^{-.6(2.3026)y_0^{-\frac{1}{2}}}.$$

Differentiating, $$dV = .69078\, V y_0^{-\frac{3}{2}} dy_0 .$$

Now, $P = \frac{VdV}{du}\,\frac{w}{\omega g}$; and $u_0 y_0 = u$, whence $du = u_0 dy_0$; also, $u_0\omega = C$, the chamber volume.

$$\therefore P = .69078 \,\frac{V^2}{C}\,\frac{w}{g}\, y_0^{-\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

This will give us the effective pressure very accurately along the chase of a gun. If C is taken in cubic feet, all other quantities being in feet and pounds, P will be per square foot, and if divided by 144 will be per square inch.

103. Recoil of Gun.—The sum of the momenta of the masses of the system moving in one direction must be equal to that in the opposite direction. At the instant when the projectile passes the muzzle of a gun, assuming that gun and carriage are unopposed in their recoil, and denoting their combined weight by W_G and their velocity by V_G, and also supposing that the charge and projectile alone constitute the remainder of the system, we have very approximately, V being the muzzle velocity of the projectile,

$$W_G\, V_G = wV + \frac{\tilde{\omega} V}{2},$$

or,

$$V_G = \frac{1}{W_G}\left(wV + \frac{\tilde{\omega} V}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (241)$$

This is less than the maximum velocity of recoil for the following reason: immediately after the projectile leaves the gun, the powder-gases escape with a high velocity and the sudden increase in their momentum is of course balanced by a like increase in that of the gun.

The gun on being fired is acted upon by pressures similar (though not the same) to those acting on the projectile; a curve of pressures on the breech face could be constructed for the travel of the gun relatively to the projectile, similar to that for the travel of the projectile (relatively to the gun). A curve of velocity of recoil could be constructed also. This latter curve, like that of the velocity of the projectile, would be convex with respect to the axis representing travel. The mean velocity of recoil while the projectile is in the gun, the curve being convex, is therefore greater than $\frac{1}{2}V_G$. As a rough approximation we may place the mean velocity of recoil equal to $\frac{6}{10}\, V_G$. The same approximation will make the mean velocity of the projectile while in the bore equal to $\frac{6}{10}V$. The time occupied by the projectile in the gun after commencing to move will then be equal to $\frac{10u}{6V}$ where u is the travel to the muzzle. While the projectile is in the gun, then, the gun recoils a distance (multiplying time by mean velocity)

$$U_G = \frac{V_G}{V}\, u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (242)$$

104. Explosions in a Block of Lead.—It has been found that the volume obtained by exploding a weight of any explosive in a block of lead is proportional to the weight used if that is very small compared with the weight of the

block. A known weight of the explosive is placed at the bottom of a cylindrical channel similar in shape to that used by miners. If necessary, the explosive is enclosed in a covering impermeable to water. The hole is then filled with water to act as tamping, the explosive is detonated by means of the electric current (and a suitable fuze if necessary), and the volume of the pear-shaped chamber produced is measured. With black and brown powders the tamping is driven out before the chamber is formed. The following are the results with the more rapid explosives according to the "Commission des Substances Explosives," one gramme being exploded (in its own volume).

Explosive, 1 gramme.	Final Volume, Cubic centimeters.
Nitro-mannite	43
Nitro-glycerin	35
Dynamite, 75 per cent.	29
Dry gun-cotton	34
Ditto (40 per cent.) + ammonium nitrate (60 per cent.)	32
Ditto (50 per cent.) + potassium nitrate (50 per cent.)	21
Mercuric fulminate	13.5
Ditto (eliminating the weight of mercury by calculation)	45

Weights of these explosives in the inverse ratio of the above volumes should, when fired in a torpedo, produce about the same result.

The volume of chamber hollowed out does not measure either the pressure or the work, but certain more complicated effects. For example, nitro-mannite and nitro-glycerin have theoretically about the same force. Nitro-mannite shows a marked tendency to tear the blocks of lead in diagonal directions. The pressure of nitro-mannite is so quickly developed that the piston of the crusher gauge is often broken, showing its shattering character.

105. Angular Velocity.—If s is the number of turns a projectile makes per second, and if we denote its *angular velocity*, that is, the angle in circular measure turned through per second, by η, we have

$$\eta = 2\pi s \qquad (243)$$

If V denote the muzzle velocity, and if n_2 denote the number of calibres it will be necessary for the projectile to travel in order to make one complete turn with the final twist given by the gun, we will have at the muzzle

$$s = \frac{V}{n_2 c} .$$

and

$$\eta = \frac{2\pi}{n_2 c} V, \qquad (244)$$

where V and c are in the same units.

106. Rotational Energy of a Projectile.—The rotational energy of a projectile is the sum total of the rotational energy of its parts.

Suppose that it is required to find the rotational energy of a cylindrical projectile. Let h be the length of the projectile, and r the radius of any one of

the elementary cylinders of thickness dr that are supposed to compose it. Let c be the outer diameter, w the weight, and supposing the projectile homogeneous, denote the mass of unit volume of it by ρ.

The volume of the cylindrical element will be $2\pi hrdr$, and its mass $2\pi\rho hrdr$. The angular velocity of rotation being η, the rotational velocity of the element will be ηr; and its kinetic energy, the half product of its mass, and velocity square will be

$$\pi\rho hrdr(\eta r)^2.$$

Integrating this, we have for the rotational energy of the cylinder

$$\pi\rho h\eta^2\int_0^{\frac{c}{2}} r^3dr = \frac{\pi\rho h\eta^2 c^4}{64} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (245)$$

The total mass of the cylinder is the sum of the masses of its cylindrical elements; that is

$$2\pi\rho h\int_0^{\frac{c}{2}} rdr = \frac{w}{g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

or,

$$\frac{\pi\rho hc^2}{4} = \frac{w}{g}.$$

Substituting in (245) we have

$$\text{Rotational energy of cylinder} = \frac{wc^2}{16g}\eta^2 . \quad . \quad . \quad . \quad . \quad (247)$$

107. Moment of Inertia.—The *moment of inertia* of a revolving body is the sum of the products of the masses of the various parts into the squares of their distances from the axis of rotation.

It is evidently equal to twice the rotational energy of the body when the angular velocity is unity. Denoting moment of inertia by I, we have

$$\text{Rotational energy} = I\frac{\eta^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

108. Radius of Gyration.—The radius of gyration of a revolving body is such a radius that if the mass of the body be supposed concentrated at its moving extremity, the rotational energy will remain unchanged. It is denoted by k.

Evidently, then, by the definition of k, we have (see also (248))

$$\text{Rotational energy} = I\frac{\eta^2}{2} = \frac{w}{2g}(k\eta)^2 . \quad . \quad . \quad . \quad . \quad (249)$$

109. Various Moments of Inertia.—Calculating the rotational energy as already shown for a cylinder, then using (248), the following values of I are found for the various shapes of projectiles, or their parts:

For a solid cylinder of diameter c and length h, $I = \rho\dfrac{\pi hc^4}{32} = \dfrac{wc^2}{8g}$.

For a hollow cylinder of diameters c and d, and length h,

$$I=\rho\frac{\pi h(c^4-d^4)}{32}=\frac{w(c^2+d^2)}{8g}.$$

For a solid sphere of diameter c, $I=\frac{\rho\pi c^5}{60}=\frac{wc^2}{10g}$.

For a hollow sphere of diameters c and d, $I=\rho\frac{\pi(c^5-d^5)}{60}=\frac{w}{10g}\cdot\frac{c^5-d^5}{c^3-d^3}$.

For a solid ogival head of σ calibres, $I=\int_0^{\frac{c}{2}\sqrt{2\sigma-1}}\rho\frac{\pi y^4dx}{2}$,

where

$$y=\sqrt{\frac{(\sigma c)^2}{4}-x^2}-\frac{c}{2}(\sigma-1).$$

For a solid ogival head of 2 calibres, $I=.036\rho\pi c^5=\frac{18}{187}\frac{wc^2}{g}$.

The moment of inertia of a projectile composed of several regular shapes is the algebraic sum of the moments of inertia of its parts.

110. **Table Calculation Simplified.**—Tables I. and II. at the end of the book have been calculated with the aid of Legendre's Tables of Elliptic Functions, using a value of n, the ratio of the specific heats of powder-gases under constant pressure and constant volume, equal to 1.4. This is the ratio for perfect gases, and it is probable that powder-gases in their highly heated state, during explosion, approach very closely to this perfect state. (At the same time it should be remembered that at ordinary temperatures this ratio is slightly less than 1.4.) As stated in a previous chapter, heat is lost to the walls of a gun during firing, and the consequent error may be allowed for approximately either by changing, in the formulæ, the value of the quantity representing the volume of the non-gaseous residue (effected by a change in y), or by changing the value of n, or by both. Unfortunately, no fixed value of n will exactly represent the conditions for different guns. As a verification however, of the functions already deduced using $n=1.4$, their values found by employing other values of n are interesting, and are shown in what follows. Only values of n that will simplify the calculation will be given.

The general expression for Y_1, obtained by replacing the exponent of y, namely 1.4, by n in (147), and deriving the expression corresponding to (149) as before, is

$$Y_1=\int_1^v\frac{dy}{y^{\frac{1}{2}}(y^{n-1}-1)^{\frac{1}{2}}}. \quad (250)$$

We will first take the case when $n=1\frac{1}{2}$. Substituting in (250), we have

$$Y_1=\int_1^v\frac{dy}{y^{\frac{1}{2}}(y^{\frac{1}{2}}-1)^{\frac{1}{2}}}.$$

In this place $y^{\frac{1}{2}} = \sec^2\varphi$; we have $y = \sec^4\varphi$, and $dy = 4\sec^4\varphi \tan \varphi d\varphi$.

$$\therefore\ Y_1 = 4\int_0^\varphi \sec^2 \varphi d\varphi = 4 \tan \varphi = 4(y^{\frac{1}{2}} - 1)^{\frac{1}{2}}. \quad . \quad . \quad . \quad (251)$$

Next, place $n = 1\frac{1}{3}$. Then, making $y = \sec^6\varphi$,

$$Y_1 = 6\int_0^\varphi \sec^3 \varphi d\varphi = 3 \ (\tan \varphi \sec \varphi + \log_e (\sec \varphi + \tan \varphi)) \quad . \quad (252)$$

(See Integral Calculus.) .

Next, let $n = 1\frac{1}{4}$. Then making $y = \sec^8 \varphi$,

$$Y_1 = 8\int_6^\varphi \sec^4 \varphi d\varphi = 8\int_0^\varphi (1 + \tan^2 \varphi) \sec^2 \varphi d\varphi = 8 \tan \varphi + \frac{8}{3} \tan^3\varphi . \quad (253)$$

Similarly, if $n = 1\frac{1}{5}$, making $y = \sec^{10} \varphi$.

$$Y_1 = 10\int_0^\varphi \sec^5 \varphi d\varphi .$$

If $n = 1\frac{1}{6}$, making $y = \sec^{12} \varphi$,

$$Y_1 = 12\int_0^\varphi \sec^6 \varphi d\varphi .$$

If $n = 1\frac{1}{7}$, making $y = \sec^{14} \varphi$,

$$Y_1 = 14\int_0^\varphi \sec^7 \varphi d\varphi ;$$

and so on.

If $n = 1\frac{1}{12}$, which is near the value found by Messrs. Noble and Abel for all the products of combustion at ordinary temperatures, we place $y = \sec^{24} \varphi$, whence

$$Y_1 = 24\int_0^\varphi \sec^{12} \varphi d\varphi .$$

All the above expressions are integrable, and their values are given in the Integral Calculus. The use of values of n varying from $1\frac{1}{2}$ to $1\frac{1}{4}$ will not very materially change the form of the velocity and pressure curves, depending as these do on the determination of two constants which serve to eliminate the errors of theory.

111. Experiments of Lt.-Col. Sebert and Captain Hugoniot.—In a "Study of the Action of Powder in a 10-Centimeter Cannon," Lieutenant-Colonel H. Sebert and Captain Hugoniot, of the French Marine Artillery, use 1.3 as the value of n, the ratio of the specific heats of powder-gases; that is, they place

$$pv^{1.3} = k.$$

They also place $P_B = P\left(1 + \frac{\tilde{\omega}}{2w}\right)$ and, calling p the mean of P_B and P,

they write

$$p = P\left(1 + \frac{\tilde{\omega}}{4w}\right) = \frac{P}{w}\left(w + \frac{\tilde{\omega}}{4}\right),$$

$$\therefore\ P\left(1 + \frac{\tilde{\omega}}{4w}\right) v^{1.3} = k\,;$$

or,

$$Pv^{1.3} = \frac{4wk}{4w + \tilde{\omega}},$$

$$\therefore\ \frac{w(V^2 - V_0^2)}{2g} = \int_{v_0}^{v} P dv = \frac{4wk}{(4w + \tilde{\omega})} \int_{v_0}^{v} \frac{dv}{v^{1.3}}\,;$$

$$\therefore\ V^2 - V_0^2 = \frac{8gk}{4w + \tilde{\omega}}\left(\frac{1}{v_0^{.3}} - \frac{1}{v^{.3}}\right),$$

where V_0 and V are velocities of the projectile corresponding to volumes v_0 and v occupied by the powder gases.

By placing v equal to infinity, whence $\frac{1}{v^{.3}} = 0$, they obtain the total work capable of being performed by the powder gases starting with the volume v_0.

These experimenters also state as a result of their experiments "that the velocity of combustion of powder is proportional to the pressure, the thickness burned being proportional to the velocity of the projectile." The pressure meant is of course the real accelerating pressure, and not what has in the present work been termed the pressure of the surrounding medium. The thickness burned is of course proportional to γ so that, according to Messrs. Sebert and Hugoniot, the square of the velocity should be proportional to γ^2; in other words, very approximately for the gun and the A_3S powder used by them

$$Y\gamma(1 - \lambda\gamma) = K\gamma^2\,;$$

(the first member of this is the second of (115) omitting constants and placing $\theta(y) = \varphi(\gamma) = a\gamma(1 - \lambda\gamma + \text{etc.})$).

Also

$$Y_0(1 - N_2 Y_1) = K_1\gamma^2 = K_2 Y_1^2$$

must hold very approximately.

112. Pressure Gauges in the Base of a Shell.—What pressure is measured by a crusher gauge in the base of a shell fired from a gun?

Suppose a cylindrical longitudinal channel bored through a shell, the diameter of the channel being the external diameter of the piston of the crusher gauge. Place the piston of the crusher gauge preceded by the copper disc in the rear end of the channel, and suppose the shell fired from a gun. The piston which we suppose absolutely free to move will, if small comparatively, move faster than the projectile, passing through the channel. When half way through, we will suppose the rear end of the projectile closed, and in

the rear part of the channel we will suppose a pressure that will give the piston the same acceleration relatively to the projectile that it had in excess before the rear end was closed. Then suppose the forward end closed, making a seat for the copper disc; the crushing will evidently correspond to the pressure that would cause the excess of acceleration (the difference between that of projectile and piston), and will evidently be the same as if the rear end of the projectile had remained open; the pressure then recorded by a crusher gauge in the rear of a projectile is evidently not the effective pressure which causes the acceleration of the piston or of the shell, but the pressure which causes the excess of acceleration of piston over shell.

If, then, the shell were a cylinder perfectly free to move, and the mass of the piston were one-sixth of the mass taken from the shell in boring the cylindrical channel, the pressure that would be recorded would be $\frac{5}{6}$ the maximum effective pressure.

If a crusher gauge were placed in the centre of the base of a shell of the same material as the gauge-piston, the length of the piston being equal to the mean length of the shell, the pressure recorded when the shell was fired from a gun would be the maximum passive resistance (per piston area) of the gun, that is, that due to forcing, back resistance of rifling, atmospheric resistance, etc. If the shell were perfectly free to move, the piston would never reach the bottom of the gauge, and if the length of the piston were greater than the mean length of the shell, it would have a backward motion relatively to the shell.

To really measure the pressure on the shell-base in this way, the piston would have to be of infinitesimal mass.

EXAMPLES.

1. The 57-mm. Hotchkiss rapid firing gun was originally designed for black powder, and for many years the French C 2 was exclusively employed. On the appearance of the brown or cocoa powders a special brand of this type, designated as brown C 2, was developed for the gun at Sevran-Livry. A further advance has now been made in the adoption of the BN 1 smokeless powder, and it will be of interest to compare the effects of these three natures of powder. (For the BN_1, See Chap. XI.)

The results obtained in the standard gun are as follows:

	C 2.	Brown C 2.
Weight of charge	$\tilde{\omega} = 0.885$ K.	0.920 K.
Weight of shell	$w = 2.720$ K.	2.720 K.
Muzzle velocity	M. V. $= 553.0$ m. s.	600.0 m. s.
Pressure at the breech	$P_M = 2550$ K.	2550 K. per (cm.)^2.

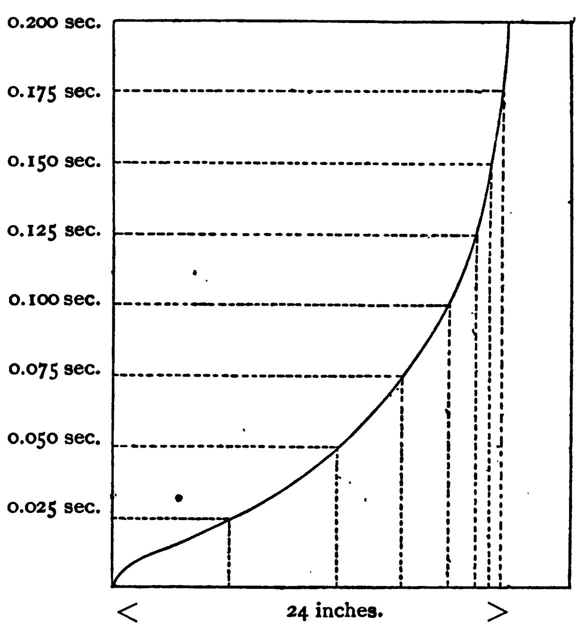
Find the mean value of N in the Gavre formula (239).

For black C 2, $N = 935.6$,
and for brown C 2, $N = 1015.1$,

whence we are able to compute the following table:

Travel of Shell.	Charge o. K 885 C 2.		Charge o. K 920 Brown C 2.	
	Velocity.	Pressure.	Velocity.	Pressure.
Decimeters.	Meters.	Kilos.	Meters.	Kilos.
28.45	595.0	269	645.6	317
20.20	553.0	364	600.0	429
17.92	538.1	402	583.8	473
15.64	521.1	446	565.3	525
13.93	506.6	486	549.6	572
12.22	490.3	532	532.0	626
10.51	471.8	585	511.9	689
8.80	450.4	649	488.7	764

2. The weight of the 6″ B. L. R. gun and top-carriage is 14,000 pounds, the projectile 100 lbs., charge 50 lbs. and the muzzle velocity is 2000 f. s. How



Development of the Cylindrical Surface of Dashiell Recoil Velocimeter.

Curve of recoil, 6-inch B. L. R. on Broadside Carriage.

long after starting is the projectile in the gun, how far does the gun recoil in the same time, and what is the velocity of recoil as the projectile leaves the gun, assuming the motion of the gun unresisted?

$t = .01$ seconds.
$U_G = 1.3$ inches.
$V_G = 18$ f. s.

3. A bullet, weight 480 grains, or .0686 lbs., is fired from a small-arm of weight 9 lbs., the charge being 80 grains, or .0114 lbs; the piece is laid on its side on a smooth board, and a straight line is drawn crosswise directly under the muzzle. In one firing the piece abuts against a tree or wall, and in a second case it is supposed perfectly free to move. How far will the tangent crosswise to the smoke-mark in the second case be in rear of that in the first as shown by reference to the original muzzle marks? The travel of bullet in piece is 32″ and the M. V. = 1400 f. s. *Ans.* .26″.

4. Assuming the products in rear of a projectile in an unchambered gun to be of uniform density, find their kinetic energy, V being the velocity of the projectile. (See Ex. 5, Chap. III.) *Ans.* $\frac{\bar{\omega} V^2}{6g}$.

5. Compare the angular velocities of rotation of a 9-inch projectile, the twist of rifling being $\frac{\pi}{45}$, with that of a 3-inch projectile, twist $\frac{\pi}{30}$, when the velocity of each is 1400 f. s. *Ans.* $\frac{2}{9}$.

6. What is the linear velocity of rotation of a point on the circumference of a 3-inch shell, the twist being $\frac{\pi}{30}$ and the M. V. 1390 f. s. *Ans.* 145.6 f. s.

7. With uniform twist, does a projectile rotate as rapidly half way down the bore as at the muzzle?

8. In the 8″ B. L. R., the velocity of the projectile is 1734 f. s. and the angle of the rifling is 4° 52′ 40″ (with an element of the bore). What is the rotational energy of the projectile, assuming $k = 3''$, the projectile weighing 250 lbs?

$$\frac{I\eta^2}{2} = \frac{250}{2 \times 32.2} (1734)^2 \left(\tan (4^\circ\ 51'\ 40'')\right)^2 \left(\tfrac{3}{4}\right)^2 \text{ foot lbs.}$$

CHAPTER XI.

SMOKELESS POWDERS.

113. **Mr. Benét on Smokeless Powders.**—In "A Study of the Effects of Smokeless Powder in a 57-mm. Gun," Mr. Laurence V. Benét, Artillery Engineer of the Hotchkiss Company, gives the results of a most thorough investigation of the subject of smokeless powders. The following is given in nearly his own words:

"The present tendency of artillery seems to be towards extremely long guns; and while a piece of thirty-five calibres length of bore was not long since considered remarkable, we now see guns of forty-five and even fifty calibres. A knowledge of the law of development of velocity in the bore is necessary to form a judgment of where it is advisable to limit this length.

The experiments about to be described were undertaken with a view of throwing some light on these questions. They consisted in cutting successive lengths from the chase of a 57-mm. gun, and observing the velocities of a series of rounds fired with each resulting travel of projectile. Subsequently a similar gun was lengthened to fifty calibres travel, and from the firings with this piece another point on the velocity curve was obtained.

The two *guns* employed in the experiments were all-steel 57-mm. Hotchkiss rapid-firing guns. Of these, one was of the standard pattern. It had been fired seventy-three rounds previous to the experiments, and was in perfect condition. The other was precisely similar in every respect, except that its chase was a few inches longer, to permit screwing on a false muzzle. The principal data of the standard gun are as follows:

Calibre,	57 mm.
Area of cross section of bore,	0.2592 dm^2.
Equivalent diameter,	0.5745 dm.
Net volume of powder chamber,	0.887 dm^3.
Total length of the piece,	2480 mm.

Total length of bore,	2280 mm.
Length of rifling,	1953.5 mm.
Angle of rifling { at beginning,	1° 0′.
Angle of rifling { at muzzle,	6° 0′.
Total weight of the piece,	360 kilos.

The gun was rifled with an increasing twist, making an angle of 1° 0′ with the axis of the bore at the origin, and of 6° 0′ at 253.5 mm. from the muzzle; for the remaining distance the twist was uniform (6° = 1 turn in 29.9 calibres). The development of the increasing portion was an arc of a circle of 19523 mm. radius.

Throughout the firings the guns were mounted on a standard Hotchkiss "*non-recoil*" mount. This carriage, through its elasticity, permits the gun to recoil about 20 mm.

The *projectiles* employed were cast iron common shell, loaded with sand. The bands were made of solid drawn and annealed brass tubing.

Weight of loaded shell,	2.720 kilos.
Principal radius of gyration,	20.8 mm.
Coefficient of form,	0.8

The *cartridge cases* were for the standard Hotchkiss built-up pattern, and require no special description. When using a crusher gauge a hole is drilled through the head of the cartridge and counter-bored from the outside to provide a seat for the copper gas check.

Two brands of the same type of *smokeless powder* were employed, both of which were manufactured at the *Poudrerie Nationale de Sevran-Livry*; they were designated as BN_1 and BN_{144}. These powders are in the form of thin strips, which are scored longitudinally on one side with a series of parallel and very narrow grooves.

In color the powder is a yellowish gray, and in texture resembles a fine-grained sandstone. Unfortunately, the chemical composition is unknown, and only the following data can be given:

	BN_1	BN_{144}
Length of strips,	76 mm.	85 mm.
Distance between scores,	1.4 mm.	1.6 mm.
Thickness of strips,	0.5 mm.	0.6 mm.
Specific gravity,	1.57	1.78

It may be added that BN_1 is the standard smokeless powder for the 57-mm. gun, and that BN_{144} was a sample submitted for trial in the same piece, but found too quick.

To form the charge the strips of powder were made up into compact bundles or fagots of such size as would enter the mouth of the cartridge case, and secured with twine. The fagots were then placed one above the other in the cartridge, the twine being removed at the moment of insertion. An igniting charge of five grams of sporting powder, contained in a thin muslin bag, was placed over the primer of the cartridge. The charge of BN_1 powder was made up of three fagots of 141 grams and one of 37 grams; that of BN_{144} was composed of two bundles of 170 grams and one of 60. The space between the top of the charge and the base of the projectile was filled with felt wads. The elements of loading were as follows:

	BN_1	BN_{144}
Weight of charge,	0.460 kilos.	0.400 kilos.
Density of loading,	0.519.	0.451.

From the firing records of the two guns we obtain the following table, giving the velocity of the projectile corresponding to each length in the bore.

Velocity in the bore with BN_1.	Travel of Shell.		Velocity in the bore with BN_{144}.
Meters.	Decimeters.	Calibres.	Meters.
682.1	28.45	49.91	632.8
648.3	20.20	35.44	600.7
636.5	17.92	31.44	591.0
622.3	15.64	27.44	573.5
612.6	13.93	24.44	565.0
595.0	12.22	21.44	553.1
574.4	10.51	18.44	534.9
543.1	8.80	15.44	503.7

Taking the means of the observations we find the maximum pressures at the breech to have been:

BN_1 powder, $P_M = 2547$ kilos. per cm.².
BN_{144} powder, $P_M = 2543$ kilos. per cm.²."

114. Empirical Formulæ for Velocity.—In the endeavor to form a curve of pressures by the differentiation of the expression for velocity, Mr. Benét worked with a number of formulæ. The various forms, together with their final numerical shape for BN_1 powder, and the probable differences, are given below. In the formulæ, y_0 is the expansion of chamber volume to the point considered, that is (see (14)),

$$y_0 = \frac{u + u_0}{u_0}.$$

	General Forms of Assumed Formulæ.	Empirical Formulæ with most probable values of the constants for BN_1 powder.	Probable Differences in metres.	
(a)	$V = Au^B$	$V = 373.5u^{.183}$	4.99	
(b)	$V = Au^{\frac{3}{16}}$	$V = 369u^{\frac{3}{16}}$	5.06	M. Sarrau's Monomial Formula.
(c)	$V \doteq Au^{\frac{3}{8}} - ABu^{\frac{7}{8}}$	$V = 251.87u^{\frac{3}{8}} - 8.0556u^{\frac{7}{8}}$	20.53	M. Sarrau's Binomial Formula.
(d)	$V = N10^{-.6y_0^{-\frac{1}{2}}}$	$V = 1115.8 \times 10^{-.6y_0^{-\frac{1}{2}}}$	8.63	M. Helie's, or Gavre Formula.
(e)	$V = A + Bu + Cu^2 + Du^3$	$V = 354.097 + 30.385u - 1.072u^2 + .0144u^3$	2.74	
(f)	$V = N10^{-Ay_0^{-\frac{1}{2}}}$	$V = 985.24 \times 10^{-.43537y_0^{-\frac{1}{2}}}$	3.89	
(g)	$V = N10^{-Ay_0^{-\frac{3}{4}}}$	$V = 845.84 \times 10^{-.48635y_0^{-\frac{3}{4}}}$	2.88	
(h)	$V = N10^{-Ay_0^{-1}}$	$V = 783.07 \times 10^{-.555y_0^{-1}}$	2.08	
(i)	$V = N10^{-Ay_0^{-\frac{5}{4}}}$	$V = 747.64 \times 10^{-.6723y_0^{-\frac{5}{4}}}$	1.78	Mr. Benét's Choice.
(j)	$V = N10^{-Ay_0^{-\frac{3}{2}}}$	$V = 724.52 \times 18^{-.842y_0^{-\frac{3}{2}}}$	2.21	

Form (*e*), if eight terms had been used, could, of course, have been made to satisfy all the results of the practice, and therefore, to reproduce them. The advantage of the exponential form involving three constants over the monomial form, involving only two, is illustrated in the accuracy of (*f*), (*g*), (*h*), (*i*), and (*j*) as compared with (*a*) and (*b*). Form (*i*), chosen by Mr. Benét, is also very accurate for smokeless powder BN_{144}.

115. Captain Ingalls' Comparison.—Using the expansion of the initial air-space, and the velocities when $u = 8.8$ and $u = 28.45$ as calculated by Mr. Benét (these being the most probable values), Captain James M. Ingalls, 1st. Artillery, U. S. Army, reduces formula (216) to the form

$$V^2 = 194295 Y_0(1 - .067657 Y_1). \quad . \quad . \quad . \quad . \quad . \quad (k)$$

The following table shows the agreement between the velocities as computed by Mr. Benét using (i), and by Captain Ingalls using (k), for smokeless powder BN_1.

Travel of Projectile.	Number of Expansions.	Velocities Computed by		Differences.
		Benét.	Ingalls.	
Decimeters.	y (see (12))	Meters.	Meters.	Meters.
28.45	13.2237	679.9	679.9	0.0
20.20	9.6786	651.0	651.4	—0.4
17.92	8.6981	639.0	638.9	+0.1
15.64	7.7187	623.9	623.3	+0.6
13.93	6.9841	610.0	609.3	+0.7
12.22	6.2495	593.1	592.3	+0.8
10.51	5.5149	572.1	571.4	+0.7
8.80	4.7803	545.5	545.5	0.0

These differences are practically *nil.*

The mean difference between measured and calculated velocities, using the first and last *measured* velocities to determine the velocity formula, is only very slightly in excess of that using (i) where the constants are determined by the method of least squares, using all the results. From the *measured* extreme velocities Captain Ingalls finds, using the expansion of the initial air-space,

$$V^2 = 189737\, Y_0(1 - .065637 Y_1). \quad . \quad . \quad . \quad (254)$$

116. Approximate Velocities and Pressures.—Without further approximation, Captain Ingalls determines the pressure formula. As might have been expected, the computed maximum pressure is a little too great. He finds,

$$P = 3399.5 X_0(1 - .027648 X_1). \quad . \quad . \quad . \quad (255)$$

For the smokeless powders employed by Mr. Benét in his experiments, according to Captain Ingalls, $a = 2$, $\lambda = \frac{1}{2}$, and $\mu = 0$. The weight of powder burned at any point is, therefore (see (235)),

$$\bar{\omega}\varphi(y) = \bar{\omega}ay(1 - \lambda y) = .120774Y_1(1 - .065637Y_1) \times 2\bar{\omega}. \quad (256)$$

It will be noticed in the following table of the results of formulæ (254) and (255), calculated by the same writer, that as the powder is completely burned (assuming results true) just short of the muzzle, it is practically a maximum one.

SMOKELESS BN_1 POWDER.

$c = 57$ mm. $\bar{\omega} = 0.46$ kg. $w = 2.72$ kg. $z = 2.32783$ dm.

Expansions $y = \frac{u+z}{z}$	Travel u decimeters.	Travel u calibres.	P kg. per cm².	V Meter seconds.	Powder burned kilograms.
1.0	0.0	0.0	0	0	0
1.1	0.233	0.408	2132	81.0	0.112
1.2	0.466	0.817	2574	129.6	.152
1.3	0.698	1.225	2741	168.9	.180
1.4	0.931	1.634	2787	200.2	.202
1.5	1.164	2.042	2770	228.0	.221
1.6	1.397	2.450	2720	252.5	.237
1.7	1.629	2.859	2651	274.3	.251
1.8	1.862	3.267	2573	294.0	.263
1.9	2.095	3.676	2491	311.9	.274
2.0	2.328	4.084	2408	328.3	.283
2.1	2.561	4.492	2325	343.4	.292
2.2	2.793	4.901	2244	357.3	.300
2.3	3.026	5.309	2166	370.3	.308
2.4	3.259	5.717	2090	382.4	.315
2.5	3.492	6.126	2018	393.7	.322
3	4.656	8.168	1702	441.3	.349
4	6.983	12.252	1254	507.2	.385
5	9.311	16.336	962	551.6	.408
6	11.639	20.420	759	583.8	.424
7	13.967	24.503	612	608.3	.436
8	16.295	28.587	500	627.5	.444
9	18.623	32.671	413	642.8	.450
10	20.950	36.755	343	655.3	.454
11	23.278	40.839	286	665.4	.457
12	25.606	44.923	239	673.8	.459
13	27.934	49.007	199	680.7	.460
13.244	28.450	50.000	191	682.1	.460

117. **Residue and Loss of Heat.**—The formulæ for velocities and pressures in guns, (206), (213), etc., are applicable in all cases where the explosive is burned, as distinguished from cases where the explosive is detonated. There is, however, more difficulty in applying the formulæ with smokeless powders, owing to the fact that the volume of solid matter in the bore is constantly diminishing, there being practically no residue. Pressures developed by firing smokeless powder in a closed vessel would probably indicate a residue of value excessive but diminishing as the sizes of the vessel and charge increase. The excessive value would be due to the fact that the loss of heat by radiation would show in the derived formula as an apparent residue. Even with black and brown powders, working back from the muzzle to the breech, using formulæ (213), etc., the calculated maximum pressure is generally a little too high. This may readily be accounted for in one of two ways. The exponent n which has been taken as 1.4 may be too large; or the assumed volume for the residue, that of the explosive itself, though less than that calculated from the experiments of Messrs. Noble and Abel, may still be too great. Both, in fact, may be true. There is no difficulty in calculating tables in which n is different from 1.4; but to use tables at all, n must be assumed constant. Practice will probably indicate at some future time whether or not 1.4 is too great a value for n; at present it is not known with certainty. The volume assumed for the residue, however, is notably greater than can be accounted for by any theory, and if we assume a value for it more nearly in accordance with that of theory, we may readily account for the maximum pressure without changing the value of n. It may be, then, that the value of $a = .44$ (see Par. 38), at the temperature of combustion, as determined by Messrs. Bunsen and Schischkoff, is not far wrong for ordinary gunpowders.

The experiments of Mr. Benét seem to indicate so considerable a loss of heat with smokeless powder BN_1, that the pressure of points along the chase varies little from what it would be with sufficiently strong gunpowder. The loss of heat has an effect that can be accounted for approximately by the assumption of a residue that does not exist, and the formulæ for velocity and pressure may be used approximately with smokeless powders in the same way as with ordinary gunpowders, as shown by Captain Ingalls.

118. Velocities.—It may be interesting to study the method of procedure in the case of no residue; that is, when the value of a, real and apparent, is assumed zero. Accurate work on this supposition is very long, consisting as it does of a series of approximations.

The same transcendentals (see Tables I. and II.) will apply to the calculation of velocities and pressures, but, owing to the change in a (see Par. 38), a new method of finding the proper number of expansions (or y in the tables) must be determined. As a first approximation, however, we may determine the constants M' and N' as already shown, using for y in the tables the expansion of the initial air-space, as heretofore, or that of the *chamber volume* to the muzzle. In the determination of M' and N', it is necessary to use dissimilar guns, but guns not too different in size, as the unequal loss of heat, which might be compensated in the formula by a separate factor for each size of gun, is not allowed for in the formulæ as they stand.

A very fair velocity formula may be found by firing in guns of the same calibre but of unequal lengths, as for example, 30 and 40 calibre guns; the formula should be correct for that calibre, but may require the use of a factor (nearly equal to unity) when applied to another. For a general working velocity formula, which, however, would not be well adapted to finding pressures, dissimilar guns of very different sizes should be chosen.

From the first approximate velocity formula deduced above, the weight of powder burned at the muzzle of each gun is found (see (235)). The chamber gas-space is then determined, supposing the weight of charge decreased by the weight burned to the muzzle, and from this, the number of expansions to the muzzle of each is determined afresh. With the new values of y, a new velocity formula is found; it should be very approximate, but, if desirable, the approximation may be carried to a third formula, and so on, working in the same way until successive approximate formulæ agree; or, work from the expansion of the chamber volume as well, taking for the weight of powder burned, the mean of the result obtained in this way and that obtained by the expansion of the initial air-space; or, we may work from the expansion of the chamber volume alone, especially when using powders near the maximum.

From the second (or third, etc.) approximate velocity formula, the weight of powder burned at a number of the various expansions in a gun is found, then the corresponding chamber gas-space z', supposing the original charge diminished in each case by this weight, and from the product of the reduced length of chamber gas-space, z', and the corresponding expansion y', the proper travel u is determined by multiplication. We will have, in each case,

$$u = y'z' - z',$$

or,

$$u = (y' - 1)z'. \qquad (257)$$

The velocity at various points in the bore may now be found by substituting in the approximate formula the velocity trancendentals corresponding to the various expansions y' (used as y).

If $\tilde{\omega}\varphi(\gamma)$ is the weight of powder burned at any point, the increase in the initial air-space to produce what we have termed the corresponding chamber gas-space is, using French units, $\frac{\tilde{\omega}\varphi(\gamma)}{\delta}$ (this being the volume occupied by that weight of powder) and we have

$$z' = z + \frac{4\tilde{\omega}}{\pi c^2 \delta}\varphi(\gamma);$$

or, to view the subject in another way, $\varphi(\gamma)$ being the portion burned, $\tilde{\omega}(1 - \varphi(\gamma))$ is the portion left, and the new density of loading is $\Delta' = \Delta(1 - \varphi(\gamma))$, whence we may find z' bythe formula $z' = u_0\left(1 - \frac{\Delta'}{\delta}\right)$. This latter value is independent of the units employed.

119. Pressures.—When $a = 0$, z' is variable.

Differentiating (257), we have

$$du = z'dy' + (y' - 1)dz' = z'dy'\left(1 + \frac{(y' - 1)dz'}{z'dy'}\right). \qquad (259)$$

The change in z' in an infinitesimal time (using French units) is evidently equal to the weight burned in the same time divided by the product of the density of powder and cross-section of the bore; that is,

$$dz' = 4\frac{d(\tilde{\omega}\varphi(\gamma))}{\pi c^2 \delta}. \qquad (260)$$

The weight of powder burned is given by (235); differentiating this, and substituting in (260), we have

$$dz' = \frac{4}{\pi C^2}\frac{\tilde{\omega}}{\delta}\frac{a}{\lambda}N_2 dY_1[1 - 2N_2Y_1];$$

$$\therefore \frac{(y'-1)dz'}{z'dy'} = \frac{4}{\pi C^2 z'}\frac{\tilde{\omega}}{\delta}\frac{a}{\lambda}N_2\frac{(y'-1)dY_1}{dy'}[1 - 2N_2Y_1]. \quad (261)$$

Placing, in equation (261),

$$(y'-1)\frac{dY_1}{dy'} = X', \quad . \quad . \quad . \quad . \quad . \quad (262)$$

substituting in (259), and this in turn in (171), we have for the effective pressure

$$P = \frac{2}{\pi C^2 z'}\frac{w}{g}\cdot\frac{dV^2}{dy'} \div \left(1 + \frac{4\tilde{\omega}}{\pi C^2 z'\delta}\frac{a}{\lambda}N_2X'(1 - 2N_2Y_1)\right),$$

or, denoting the pressure obtained by treating z' as a constant in the differentiation of the approximate velocity formula by P_G,

$$P = P_G \div \left(1 + \frac{4\tilde{\omega}}{\pi C^2 z'\delta}\frac{a}{\lambda}N_2X'(1 - 2N_2Y_1)\right). \quad . \quad (263)$$

Substituting Δ for its value in French units, or $\frac{4\tilde{\omega}}{\pi C^2 u_0}$, we have

$$P = P_G \div \left(1 + \frac{u_0\Delta a}{z'\delta\lambda}N_2X'(1 - 2N_2Y_1)\right). \quad . \quad (264)$$

This simply requires the ratio of u_0 to z', and is independent of the units.*

The logarithms of X' for different values of y' (y in tables) are given in Table II. They were calculated from the formula

$$X' = (y-1)y^{-.7}Y^{-.5}, \quad . \quad . \quad . \quad . \quad (265)$$

obtained by substituting for $\frac{dY_1}{dy}$, in (262), its value $Y^{-.5}y^{-.7}$ (see (223)).

From the results of a number of other firings with smokeless powders, it seems approximate to say that the muzzle velocity varies as the $\frac{3}{4}$ power of the weight of charge for small changes

* Methods similar to those used in the present and preceding paragraphs may be employed for *any* value of a.

from working conditions. The variation in the power corresponding to the change in pressure is very great, extending from about $1\frac{1}{4}$ to about $2\frac{3}{4}$. As a very rough approximation, we might say that the pressure varies as the square of the charge. The data published in regard to the experiments are generally too meagre for a close observation of the effect of other changes.

Examples.

1. Using the second and last values of the velocity in the bore with BN_1 powder (see Paragraph 113), find a first approximate velocity formula.

We have

$$(648.3)^2 = [\lg^{-1}.59987]M_2(1 - N_2[\lg^{-1}.82415])$$

and $$(543.1) = [\lg^{-1}.36221]M_2(1 - N_2[\lg^{-1}.69472]).$$

Whence, dividing the equations member by member, eliminating M_2, clearing of fractions and transposing,

$$N_2 - [\lg^{-1} 8.83149] = .06784\,;$$

and substituting this in either of the above, and dividing by the coefficient, we have

$$M_2 = [\lg^{-1} 5.28533] = 192900\,;$$

$$\therefore\ V^2 = 192900\, Y_0(1 - .06784\, Y_1)\,;$$

or, $$V^2 = [\lg^{-1} 5.28533]\, Y_0(1 - [\lg^{-1} 8.83149]\, Y_1).$$

This is the first approximate formula, and may be used for differentiation, treating z as a constant, to find the pressure.

2. Assuming $a = 2$, $\lambda = \frac{1}{2}$ and $\mu = 0$, find the portion of charge burned and z' in the two cases, assuming no residue, real or apparent.

We have $$\frac{a}{\lambda} = 4, \text{ and } \log\frac{a}{\lambda} = .60206\,;$$

adding this to the second logarithm in the value of V^2 above, we have for the portion of charge burned (see (234)),

$$\varphi(y) = [\lg^{-1} 9.43355]\, Y_1(1 - [\lg^{-1} 8.83149]\, Y_1)\,;$$

whence, when

$y = 9.7$, $\varphi(\gamma) = .99112$ and $\Delta' = (1 - .99112)\Delta = .00882 \times .519$, and when

$$y = 4.8,\ \varphi(\gamma) = .89314,\ \text{and}\ \Delta' = .10686 \times .519.$$

Hence, when

$$y = 9.7,\ z' = u_0\left(1 - \frac{\Delta'}{\delta}\right) = 3.422 \times .99706 = 3.412 \therefore y' = 6.9202$$

(see (257)), and when

$$y = 4.8,\ z' = 3.3011 \ \therefore\ y' = 3.6657.$$

3. Find the second approximate velocity formula, assuming no residue, real or apparent.

The values to be taken for y are those of y' in the last example.

We have $\quad (648.3)^2 = [\lg^{-1}.49732]M_2(1 - N_2[\lg^{-1}.76595])$,

and $\quad (543.1)^2 = [\lg^{-1}.24452]M_2(1 - N_2[\lg^{-1}.63681])$;

whence, finding the values of M_2 and N_2,

$$V^2 = [\lg^{-1}5.42622]\, Y_0(1 - [\lg^{-1}8.93197]\, Y_1).$$

4. Find the pressure when $y' = 1.4$, leaving out of consideration the fact that z is a variable.

We find, using the last equation for V^2, for the portion of charge burned,

$$\varphi(\gamma) = [\lg^{-1}9.53403]\, Y_1(1 - [\lg^{-1}8.93197]\, Y_1),$$

where $\quad Y_1 = [\lg^{-1}.28330]; \therefore \varphi(\gamma) = .54884$;

$$\therefore \frac{\Delta'}{\delta} = (1 - \varphi(\gamma))\frac{\Delta}{\delta} = .14914;$$

$$\therefore z' = u_0\left(1 - \frac{\Delta'}{\delta}\right) = \frac{.887}{.2592}(1 - .14914) = 2.9118;$$

$$\therefore P_G = \frac{2.72}{.2592 \times 2}\frac{1}{gz'}\frac{dV^2}{dy'},$$

and, substituting $g = 98.09$ (decimeters), we have

$$P_G = \frac{2.72}{.2592 \times 2 \times 98.09 \times 2.9118}[\lg^{-1}5.42622]X_0 \\ (1 - [\lg^{-1}8.93197X_1]),$$

or, $\quad P_G = 2887.6$ kilos. per (c. m.)^2.

5. Find P under the same circumstances.

$$P = P_G \div \left(1 + \frac{.46[\lg^{-1} 9.53403][\lg^{-1} 9.94967]}{2.592 \times 1.57[\lg^{-1} .46415]}\right.$$

$$\left.(1 = [\lg^{-1} 9.51628])\right) = \frac{P_G}{1.078};$$

$$\therefore P = 2679 \text{ kilos. per (c. m.)}^2.$$

6. Find the pressure when $y' = 1.2$, leaving out of consideration the fact that z' is varying.

$$\varphi(\gamma) = [\lg^{-1} 8.93197][\lg^{-1} .60206][\lg^{-1} .14111])$$

$$(1 - [\lg^{-1} 8.93197][\lg^{-1} .14111]);$$

$$\therefore \varphi(\gamma) = .4153 \text{ and } 1 - \varphi(\gamma) = .5847,$$

$$\Delta' = \Delta \times .5847 = .519 \times .5857 = .30346,$$

$$z' = \frac{.887}{.2592}\left(1 - \frac{.30346}{1.57}\right) = 2.7606,$$

$$P_G = \frac{1}{2 \times .2592} \cdot \frac{2.72}{98.09 \times 2.7606}[\lg^{-1} 5.42622]X_0$$

$$(1 - [\lg^{-1} 8.93197]X_1).$$

$$\therefore P_G = 2875.7 \text{ kilos. per (c. m.)}^2.$$

7. Find P from the last value of P_G at $y' = 1.2$.

$$dz' = \frac{\tilde{\omega}}{\omega\delta} d\varphi(\gamma) = \frac{.46}{.2592 \times .157}[\lg^{-1} 9.53403]dY_1$$

$$(1 - [\lg^{-1} 8.93197] 2 Y_1)$$

$$\frac{(y' - 1)dz'}{z'dy} = \frac{.46}{.2592 \times 1.57}[\lg^{-1} 9.53403]\frac{X'}{2.7606}$$

$$(1 - [\lg^{-1} 9.23300] Y_1) = .07096;$$

$$\therefore P = \frac{2875.7}{1.071} = 2685 \text{ kilos. per (c. m.)}^2.$$

It thus appears that the maximum pressure occurs at a value of y' of 1.2 (nearly). By using the second approximate formula, it appears that the calculated value of the maximum effective pressure has been reduced by about 100 kilos. (compare with the table calculated by Captain Ingalls). A third approximation would give a like reduction, but the value of a and λ must be known for a certainty (and for the way that the powder actually burns in the gun), to get the best results. The above pressure is about 200 kilos. too

great, the measured maximum pressure (breech block), being 2550 kilos.

8. Apply formula (i) to the results obtained with BN_{144} powder (see Paragraph 114).

Substituting in (i) the corresponding values of V and u for each round, we obtain a series of values of N. Taking their mean, we have for (i) numerically,

$$V = 692.98 \times 10^{-.6723 y_0^{-\frac{5}{4}}} = 692.98 e^{-.6723(2.3016) y_0^{-\frac{5}{4}}};$$

$$\therefore\ dV = 1.93506\, V y_0^{-\frac{9}{4}} dy_0$$

and $$P = \frac{V dV}{C} \frac{w}{g dy_0} = 1.93506 \frac{V^2}{C} \frac{w}{g} y_0^{-\frac{9}{4}},$$

where C is the the chamber volume and g gravity. We find

Travel of shell in decimeters.	Velocities computed in meters.	Differences from observed in meters.	Pressures computed in kilos.
28.45	630.1	—2.7	159
20.20	603.5	2.8	285
17.92	592.3	1.3	345
15.64	578.3	4.8	425
13.93	565.4	0.4	503
12.22	549.7	—3.4	599
10.51	530.2	—4.7	723
8.80	505.6	1.9	882

CHAPTER XII.

RIFLING, EFFECTS ON PRESSURE.

120. Resistance Due to Rifling.—In a rifled gun, let AB represent the driving edge of a groove inclined at point B, at an angle φ to an element of the bore (which is a line in the surface of the bore parallel to its axis). Let p be the pressure, perpendicular to the groove, between the gun and the rotating band of the projectile. This pressure is represented in Fig. 5 by BD. The effect is to diminish the accelerating pressure, supposing the projectile to move to the right, by

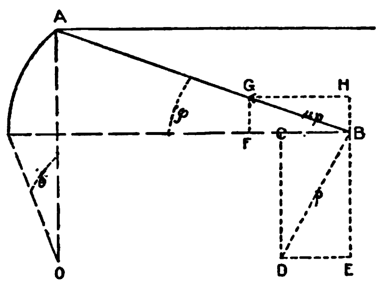


FIG. 5.

$$BC = p \sin \varphi.$$

If the coefficient of friction is denoted by μ, there is a retarding pressure along the groove of

$$BG = \mu p.$$

The effect of this is to diminish the accelerating pressure by

$$BF = \mu p \cos \varphi.$$

The total diminution is

$$BC + BF = p \sin \varphi + \mu p \cos \varphi.$$

If ωP_1 represents the total pressure on the base of the projectile, and if, as heretofore, ωP represents the total effective pressure, we have then, supposing that the rifling alone produces the difference,

$$\omega P = \omega P_1 - p(\sin \varphi + \mu \cos \varphi); \quad . \quad . \quad . \quad (266)$$

or (see (101)), $\dfrac{w}{g}\dfrac{d^2u}{dt^2} = \omega P_1 - p(\sin \varphi + \mu \cos \varphi). \quad . \quad . \quad (267)$

121. Rotating Couple.—The pressure directly causing rotation of the projectile will evidently be

$$BE - BH = p(\cos\varphi - \mu\sin\varphi).$$

This pressure is applied at the end of an arm $\frac{c}{2}$. Denoting the radius of gyration of the projectile by k, θ being the angle turned through, $k\frac{d^2\theta}{dt^2}$ is the rotatory acceleration of the extremity of the radius of gyration, $\frac{w}{g}k\frac{d^2\theta}{dt^2}$ is the force which if applied there (at that radius) would produce that acceleration in the mass $\frac{w}{g}$ supposed concentrated there, and $\frac{2k}{c}\cdot\frac{w}{g}k\frac{d^2\theta}{dt^2}$ is the force or pressure which if applied at the end of a radius $\frac{c}{2}$ would accomplish the same acceleration. This last is the pressure directly causing rotation. We may then write from the last equation,

$$\frac{2}{c}wk^2\frac{d^2\theta}{dt^2} = p(\cos\varphi - \mu\sin\varphi),$$

or,

$$\frac{wk^2}{g}\frac{d^2\theta}{dt^2} = \frac{pc}{2}(\cos\varphi - \mu\sin\varphi) \quad . \quad . \quad . \quad . \qquad (268)$$

122. Application of Formulæ.—In formula (268), we know w and g and can find k and $\frac{d^2\theta}{dt^2}$, the radius of gyration of the projectile, and the angular acceleration respectively. To find the latter, of course, we must know the acceleration of the projectile along the bore, as the angular acceleration will be proportional to it if the twist is uniform; in the case of variable twist we must know the velocity also. In (268) we will then know everything except p. We may then solve (268) for p and substitute this in (266); when, if P_1 is known, we can find P, and *vice versa*. The operation may be shortened as follows: Place in the following fraction,

$$\mu = \tan\Psi.$$

Then, dividing numerator and denominator by $\cos\varphi$,

$$\frac{\sin\varphi+\mu\cos\varphi}{\cos\varphi-\mu\sin\varphi}=\frac{\tan\varphi+\tan\Psi}{1-\tan\varphi\tan\Psi}.$$

This latter is the tangent of $(\varphi+\Psi)$. Substituting for Ψ in the last equation its value, $\tan^{-1}\mu$, and multiplying both members by $p(\cos\varphi-\sin\varphi)$, we will have

$$p(\sin\varphi+\mu\cos\varphi)=p(\cos\varphi-\mu\sin\varphi)\tan(\varphi+\tan^{-1}\mu);$$

consequently, by (268),

$$p(\sin\varphi+\mu\cos\varphi)=\frac{2w}{gc}k^2\frac{d^2\theta}{dt^2}\tan(\varphi+\tan^{-1}\mu), \qquad (269)$$

and by (266),

$$\omega(P_1-P)=\frac{2w}{gc}k^2\frac{d^2\theta}{dt^2}\tan(\varphi+\tan^{-1}\mu). \quad . \quad . \quad (270)$$

This is the difference due to rifling between the total pressure on the base of the projectile and the total accelerating pressure at any point.

123. Uniform Twist.—If we suppose the surface of the bore of a gun to be developed, that is, rolled out as a plane surface, the driving edge of a groove will in the case of uniform twist develop into a straight line. Assuming at the beginning of rifling that θ equals 0, denoting distances measured along the bore from the beginning of rifling by x, we have at any point of the edge, in the case of uniform twist (see Figs. 5 and 6), $\frac{c\theta}{2}=x\tan\varphi$; or, placing $\tan\varphi=b$,

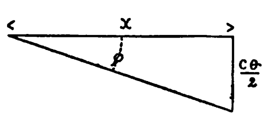


FIG. 6.

$$\frac{c\theta}{2}=bx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (271)$$

Evidently, also, since the change in the distance from the beginning of rifling is the same as the increase in the travel,

$$\frac{dx}{dt}=\frac{du}{dt}, \text{ and } \frac{d^2x}{dt^2}=\frac{d^2u}{dt^2}. \quad . \quad . \quad . \quad . \quad (272)$$

Differentiating (271) twice, remembering (272), and dividing through by $\frac{c}{2}$, we have

$$\frac{d^2\theta}{dt^2} = \frac{2b}{c} \cdot \frac{d^2u}{dt^2},$$

or, since (see (101)),
$$\frac{d^2u}{dt^2} = \frac{g}{w} \omega P,$$

$$\frac{d^2\theta}{dt} = \frac{2b}{c} \cdot \frac{g}{w} \omega P,$$

whence, by (270),

$$\omega(P_1 - P) = \frac{4bk^2}{c^2} \omega P \tan(\varphi + \tan^{-1}\mu); \quad . \quad . \quad (273)$$

or, dividing by ω, and transposing P,

$$P_1 = P\left(1 + \frac{4bk^2}{c^2} \tan(\varphi + \tan^{-1}\mu)\right),$$

or, since $\tan\Psi = \mu$, $\tan\varphi = b$, and $\tan(\varphi + \psi) = \frac{\tan\varphi + \tan\Psi}{1 - \tan\varphi \tan\psi}$,

$$P_1 = P\left(1 + \frac{4bk^2}{c^2} \cdot \frac{b + \mu}{1 - b\mu}\right). \quad . \quad . \quad (274)$$

124. Increasing Twist, Semi-Cubical Parabola.—The surface of the bore being developed and the grooves having such a form as to develop into a semi-cubical parabola, we have

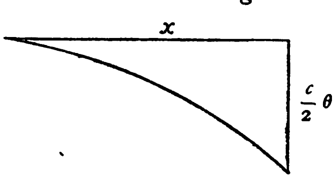


FIG. 7.

$$\frac{c}{2}\theta = bx^{\frac{3}{2}}, \quad . \quad . \quad (275)$$

where b is some constant.

Differentiate twice, as before; we have on first differentiation, dividing by dt,

$$\frac{c}{2} \frac{d\theta}{dt} = \frac{3}{2} bx^{\frac{1}{2}} \frac{dx}{dt};$$

or, remembering that
$$\frac{dx}{dt} = \frac{du}{dt} = V,$$

we have
$$\frac{c}{2} \frac{d\theta}{dt} = \frac{3}{2} bx^{\frac{1}{2}} V;$$

on second differentiation, dividing by dt,

$$\frac{c}{2}\frac{d^2\theta}{dt^2} = \frac{3}{2} b x^{\frac{1}{2}} \frac{dV}{dt} + \frac{3}{4}\frac{b}{x^{\frac{1}{2}}} V^2,$$

whence, substituting for $\frac{d^2\theta}{dt^2}$ in (270) its value from this, we have

$$\omega (P_1 - P) = \frac{6bwk^2}{gc^2}\left(x^{\frac{1}{2}}\frac{dV}{dt} + V^2\frac{1}{2x^{\frac{1}{2}}}\right)\tan(\varphi + \tan^{-1}\mu),$$

or, placing (see 101)) $\frac{dV}{dt} = \frac{d^2u}{dt^2} = \frac{g\omega}{w}P$,

and transposing ωP, we have

$$\omega P_1 = \omega P + \frac{6bwk^2}{gc^2}\left(\omega P\frac{gx^{\frac{1}{2}}}{w} + V^2\frac{1}{2x^{\frac{1}{2}}}\right)\tan(\varphi + \tan^{-1}\mu). \quad (276)$$

If P is given in pounds per square inch, ω will, of course, be in square inches. To avoid confusion, using English units, all other data should be in feet and pounds.

If z represents the variable $\tan\varphi$, (276) may be written

$$\omega P_1 = \omega P + \frac{6bwk^2}{gc^2}\left(\omega P\frac{gx^{\frac{1}{2}}}{w} + V^2\frac{1}{2x^{\frac{1}{2}}}\right)\frac{z+\mu}{1 - z\mu}. \quad (277)$$

125. Origin of Rifling.—The form of Equation (275) shows that x is measured from an origin at which the angle between the groove and an element of the bore is zero. For, denoting by y the ordinates of the developed curve corresponding to abscissa x, that is, substituting y for $\frac{c}{2}\theta$, we have

$$y = bx^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

whence $$\frac{dy}{dx} = \frac{3}{2}bx^{\frac{1}{2}};$$

or, since φ is the angle between the groove and an element (the angle remains the same in the development as in the gun),

$$\tan\varphi = \frac{3}{2}bx^{\frac{1}{2}}.$$

Consequently, when $x = 0$, $\varphi = 0$.

The point where the rotating band brings up is sometimes forward of this origin, the lands being cut away for a few inches. As the rotating band is at the real beginning of the rifling when the travel of the projectile is zero, we may always find x by adding to the travel, u, the distance from the above origin to the beginning of the rifling.

126. Rifling of Navy B. L. R.—This particular kind of rifling (semi-cubical parabola) is used in all the new Navy breech-loading rifles. In the developed curve of rifling for the new guns, the rifling generally begins with a twist of one turn in a certain number of calibres, and in a known distance depending upon the length of the gun, increases to a twist of one turn in a certain other number of calibres, this last being determined by the desired velocity of rotation.

Suppose that the rifling begins with a twist of one turn in n_1 calibres, and in a distance of d increases to a twist of one turn in n_2 calibres. Denote by x_0, the distance that the *beginning* is forward of the *origin* of rifling. The equation to the developed curve is

$$y = bx^{\frac{3}{2}}.$$

Differentiating,
$$\frac{dy}{dx} = \frac{3}{2} bx^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

When
$$x = x_0, \ \frac{dy}{dx} = \tan \varphi = \frac{\pi}{n_1}$$

(the rifling turning through a distance πc in advancing a distance $n_1 c$).

When
$$x = (x_0 + d), \ \frac{dy}{dx} = \tan \varphi = \frac{\pi}{n_2} \cdot$$

We may then write,

$$\frac{3}{2} bx_0^{\frac{1}{2}} = \frac{\pi}{n_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (280)$$

and
$$\frac{3}{2} b (x_0 + d)^{\frac{1}{2}} = \frac{\pi}{n_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (281)$$

Dividing (281) by (280), member by member, and clearing of fractions,

$$n_2(x_0 + d)^{\frac{1}{2}} = n_1 x_0^{\frac{1}{2}};$$

squaring and transposing,

$$(n_1^2 - n_2^2)x_0 = n_2^2 d:$$

$$\therefore \ x_0 = \frac{n_2^2 d}{n_1^2 - n_2^2} \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad (282)$$

From (280), we have

$$b = \frac{2\pi}{3n_1 x_0^{\frac{1}{2}}};$$

$$\therefore \quad b = \frac{2\pi}{3d^{\frac{1}{2}}} \cdot \frac{(n_1^2 - n_2^2)^{\frac{1}{2}}}{n_1 n_2}. \quad . \quad . \quad . \quad . \quad (283)$$

127. Mean Value of Pressure (Component) Causing Rotation.—The variable value of the pressure directly causing rotation is (see (268))

$$\frac{wk^2}{g}\frac{d^2\theta}{dt^2} = \frac{pc}{2}(\cos\varphi - \mu\sin\varphi).$$

The rotational energy at the muzzle is (see (249) and (244)),

$$\frac{w}{2g}(k\eta)^2 = \frac{2w}{g}\left(\frac{\pi k}{cn_2}\right)^2 V^2. \quad . \quad . \quad . \quad (284)$$

Let l be the distance the projectile has turned through from seat to muzzle. l is evidently the difference of ordinates of the developed curve of rifling, of which the equation is

$$y = bx^{\frac{3}{2}},$$

at $x = x_0$, and $x = x_0 + d$, the distance d ending at the muzzle; if the character of the rifling changes inside of the muzzle the increase in the ordinate, according to the new rule, must be found from the point of change to the muzzle.

The rotational energy of the projectile is produced by the mean pressure directly causing rotation acting through distance l. This mean value, therefore, is (since work or resulting energy is the product of the mean force by the distance through which it acts)

$$\frac{w}{2gl}(k\eta)^2 = \frac{2w}{gl}\left(\frac{\pi k}{n_2 c}\right)^2 V^2; \quad . \quad . \quad . \quad (285)$$

or, in terms of the angle turned through, denoting the angle by α and remembering that

$$l = \frac{c}{2}\alpha,$$

we have, for the mean value of the pressure directly causing rotation,

$$\frac{w}{gc\alpha}(k\eta)^2 = \frac{4w}{gc\alpha}\left(\frac{\pi k}{n_2 c}\right)^2 V^2. \quad . \quad . \quad . \quad . \quad (286)$$

128. Maximum Safe Turning Pressure.—The maximum safe pressure in the couple turning a projectile with respect to the gun may be determined in the following manner: Let S be the elastic strength of the metal, of which the gun is made, to resist torsional shearing stress, and R_2 and R_1 the external and internal radii of the gun at the point considered. Suppose that the gun has been so twisted about its axis that its external layer is just at the elastic limit; then, since each layer is stretched through a length proportional to its radius r, the tangential pull resisting the rotating couple at any point will be

$$S\frac{r}{R_2}; \quad \ldots \ldots \ldots \ldots \quad (287)$$

the moment of this force is

$$S\frac{r^2}{R_2},$$

and the elemental resistance to torsion is

$$2\pi r \,.\, S\frac{r^2}{R_2} \,.\, dr.$$

Since the pressure p_1 causing rotation acts with arm $2R_1$, we have

$$p_1 \,.\, 2R_1 = \int_{R_1}^{R_2} 2\pi r \,.\, S \,.\, \frac{r^2}{R_2}\, dr;$$

integrating,

$$p_1 \,.\, 2R_1 = \frac{2\pi S}{R_2} \,.\, \frac{R_2^4 - R_1^4}{4};$$

hence

$$p_1 = \frac{\pi S\left(R_2^4 - R_1^4\right)}{4 \,.\, R_2 R_1} \quad \ldots \ldots \ldots \quad (288)$$

129. Safe Turns as Regards Rotating Band.—To find the number of turns per second that a projectile may make without danger of throwing off the rotating band, supposing this to be held on only by its strength, we proceed as follows: Let R be the outside radius of the band, t its thickness, l its length (parallel to the longer axis of the projectile), ρ its mass per unit volume, η its angular velocity (see 143)), and T its strength (either elastic or ultimate).

The acceleration along the radius, at any point of the variable radius r is, V being the linear velocity of rotation of a point at that radius,

$$\frac{V^2}{r} = \frac{(\eta r)^2}{r} = \eta^2 \,.\, r = (2\pi s)^2 r,$$

s being the number of turns made per second. Let θ be the angle between any plane of rupture and that passing through the axis and the point considered. Then, calling dv the element of volume, the equation of equilibrium is

$$2tlT = 4\pi^2 s^2 r \rho \Sigma \sin\theta dv.$$

If the mass is homogeneous, $\rho dv = l\rho \,.\, rdr \,.\, d\theta$; hence

$$ltT = 4\pi^2 s^2 \rho l \int_{R-t}^{R} \int_0^{\frac{\pi}{2}} r \sin\theta \,.\, dr \,.\, d\theta \,;$$

integrating,

$$tT = 4\pi^2 s^2 \rho \,.\, \frac{R^3 - (R-t)^3}{3}, \quad \ldots \ldots \quad (289)$$

and

$$s = \left(\frac{3Tt}{4\pi^2 \rho (3R^2 - 3Rt + t^2)} \right) \quad \ldots \ldots \quad (290)$$

This formula is applicable only when t is small as compared with R.

Examples.

1. In the 8″ B. L. R. the rifling begins at the forward end of the compression slope with a twist of one turn in 180 calibres, and in a distance of 185″, or 15.083′, it increases to one turn in 30 calibres; thence to the muzzle, 13.16′, the twist is uniform. Determine the equation of the developed groove.

Ans. $x_0 = .43095'$,
$b = .017724$,
and in feet, $y = .017724 x^{\frac{3}{2}}$.

For the uniform twist, the origin being at the end of the increasing twist,

$$y = \frac{\pi}{30} x.$$

2. What is the angle of the rifling at a point where $u = 9.8'$?

Ans. $x = 9.8' + .43' = 10.23$,

$$\tan\varphi = \frac{dy}{dx} = \frac{3}{2}(.017724) x^{\frac{1}{2}},$$

$$\therefore \; \varphi = 4^\circ \, 52' \, 40''.$$

3. At the point mentioned in Example 2, the velocity of projectile is 1734 f. s., and the effective pressure is 15768 lbs. Assuming $\mu = .2$, what is the excess of the pressure upon the base of the projectile over the effective pressure, assuming $k = 3''$?

Ans. $P_1 - P = 267$ lbs. (approx.).

4. The 6-inch. B. L. R. is 193.53 inches long from breech to muzzle. The twist of rifling begins with one turn in 180 calibres, and, in a length of 134 inches along the axis of the bore, increases to

one turn in 30 calibres, thence to the muzzle, a distance of 9.85 inches, the twist is uniform or $\frac{\pi}{30}$. Deduce equations to the developed groove which will allow a rifling bar to be constructed, and find the distance of the origin of the curve from the beginning of rifling; the form of the equation for increasing twist being $y^2 = ax^3$.

Ans. $y = .005945x^{\frac{3}{2}}$,

and $y = \frac{\pi}{30}x$.

$x_0 = 3.83''$.

5. In a Woolwich 10-inch gun the length of the rifled part of the bore is 118 inches, and the twist increases from one turn in 100 calibres at the beginning to one turn in 40 calibres at the muzzle. At what point is the twist one turn in 60 calibres, the form of the developed groove being a common parabola (uniformly increasing)? The equation is $y = ax + bx^2$.

Ans. 65.56 inches from the muzzle.

6. A Hotchkiss 57-mm. R. F. Gun is rifled with an increasing twist making an angle of 1° with the axis of the bore at the beginning of rifling and of 6° at 253.5 mm. from the muzzle; for the remaining distance the twist is uniform ($6^\circ = 1$ turn in 29.9 calibres). The total length of rifling is 1953.5 mm. and the development of the increasing portion is the arc of a circle. What is the equation of the latter and its radius?

Ans. $x^2 = 2ry - y^2$.

$1700 \sec 3^\circ 30' = 2r \sin 2^\circ 30' \therefore r = 19523$ mm.

7. At the forward end of the compression slope of an 8″ B. L. R. the ordinate of the developed curve of rifling is $y = .06''$ and at the muzzle $y = 14.375''$. The surface of the projectile will therefore turn through $14.315'' = 1.193'$ in the gun. The muzzle velocity of the projectile weight 250 lbs. being assumed 2020 f. s. what is the mean value of the force that acts directly to produce rotation, the radius of gyration being assumed 3″, the final twist being one turn in 30 calibers?

The projectile turns at muzzle 101 times per second.

The velocity of extremity of radius of gyration in feet is

$$\frac{101 \times 6\pi}{12} = \frac{101}{2}\pi.$$

The rotational energy is $\dfrac{250}{2 \times 32.2}\left(\dfrac{101\pi}{2}\right)^2$.

The required force is $\dfrac{250}{2 \times 32.2 \times 1.193}\left(\dfrac{101\pi}{2}\right)^2$.

8. What is the greatest number of turns per second that a brass rotating band ¼ inch in thickness on an 8″ B. L. R. projectile may make without danger of being thrown off, assuming $T = 19,000$ lbs., and $\rho = .0098$? *Ans.* 113 (approx.).

9. The 8″ B. L. R. projectile after traveling 9.8 feet, is at a point where the external radius is 7.5″. Assuming the shearing limit 15 tons, what is the maximum pressure causing rotation (in the plane of a circle in the rotating band) that the gun can stand?

Characteristics.

The powder characteristics (see 184) and (185)) may be used in formulæ (206) and (213) in the same way as was explained in Chapter VIII. for M. Sarrau's formulæ. For very accurate work, a third characteristic would have to be assumed, namely

$$\epsilon = \frac{\mu}{\tau_2}.$$

The three characteristics could be tabulated very readily, working from any given standard powder as before. For pierced cylindrical grains, the third characteristic would be 0, and any reliable powder composed of grains of this form or of pierced prisms, such as German cocoa powder, would make a convenient standard. The exact value of N, with other shapes of grains, would be determined by solving the quadratic equation obtained by eliminating M in the velocity formulæ given by the two firings (using the proper positive root).

TABLES.

(The Tables were calculated with the aid of five-place logarithm tables, and the last figure is not always exact.)

FORMULÆ TO BE USED WITH TABLES I. AND II.

For grains of the pierced cylinder class,

$$V^2 = M' \frac{\tilde{\omega}}{c} \left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0 \left(1 - N' \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right),$$

$$P = \frac{2}{\pi c^2} \cdot \frac{w}{gz} M' \frac{\tilde{\omega}}{c} \left(\frac{z}{w}\right)^{\frac{1}{2}} X_0 \left(1 - N' \frac{(wz)^{\frac{1}{2}}}{c} X_1\right).$$

The first being reduced to

$$V^2 = M_2 Y_0 (1 - N_2 Y_1),$$

the second becomes $\quad P = \frac{2}{\pi c^2} \frac{w}{gz} M_2 X_0 (1 - N_2 X_1).$

For grains of the sphere or cube class,

$$V = M_1 \left(\frac{\tilde{\omega}}{c}\right)^{\frac{1}{2}} \left(\frac{z}{w}\right)^{\frac{1}{2}} Y_0^{\frac{1}{2}} \left(1 - N_1 \frac{(wz)^{\frac{1}{2}}}{c} Y_1\right),$$

$$P = \frac{2}{\pi c^2} \cdot \frac{w}{gz} M_1^2 \frac{\tilde{\omega}}{c} \left(\frac{z}{w}\right)^{\frac{1}{2}} X_0 \left(1 - 2N_1 \frac{(wz)^{\frac{1}{2}}}{c} X_1 + N_1^2 \frac{wz}{c^2} X_2\right),$$

or, $\quad V = M_2^{\frac{1}{2}} Y_0^{\frac{1}{2}} (1 - N_2 Y_1),$

and $\quad P = \frac{2}{\pi c^2} \frac{w}{gz} M_2 X_0 (1 - 2N_2 X_1 + N_2^2 X_2).$

For quick powders,

$V^2 = H_1 \frac{\tilde{\omega}}{w} Y$, and $P = \frac{2}{\pi c^2} \cdot \frac{w}{gz} \cdot H_1 \frac{\tilde{\omega}}{w} X$; or $V^2 = H_2 Y$, and $P = \frac{2}{\pi c^2} \frac{w}{gz} H_2 X$.

For smokeless powders (after approximating sufficiently), by means of the velocity formula and the weight of powder burned which will furnish the chamber gas-space, P_G is calculated. Its form corresponds with the above, being calculated from the velocity formula (see for grains of pierced cylinder class). From P_G the effective pressure is calculated by (see (264))

$$P = P_G \div \left(1 + \frac{u_0 \Delta a}{z' \delta \lambda} N_2 X' (1 - 2 N_2 Y_1)\right).$$

In any case, find z' in the same units as u_0 by the formula

$$z' = u_0 \left(1 - \frac{\Delta'}{\delta}\right).$$

In accurate interpolation, use the expression

$$t\left(d_1 + \frac{t-1}{2} d_2\right),$$

where d_1 is the first difference between successive tabular functions enclosing the function required, d_2 the mean of the two second differences of the four tabular functions two on each side of the one desired, and t the fraction of the difference between the two tabular values of y enclosing the value of y in question. Thus: required log Y, for $y = 17.6$.

16	9.82616		
		.00509	
17	9.83125		— .00045
		.00464	
18	9.83569		— .00040
		.00424	
19	9.84013		

For $y = 17.6$,

$$\log Y_9 = 9.83125 + .6\left(464 + \frac{.6 - 1.0}{2} (= 43)\right) \frac{1}{100000}$$

$$= 9.93115 + .6(00473) = 9.83409.$$

TABLE I.

Velocities in Guns.

Expansions.	Slow Powders.		Quick Powders.
y	$\log Y_0$	$\log Y_1$	$\log Y + 10$
1.1	8.56808—10	9.99498—10	8.57310
1.2	8.98823 "	0.14111	8.84714
1.3	9.22322 "	0.22484	8.99838
1.4	9.38341 "	0.28330	9.10010
1.5	9.50319 "	.32792	9.17528
1.6	9.59686 "	.36390	9.22396
1.7	9.67549 "	.39391	9.28158
1.8	9.74087 "	.41965	9.32122
1.9	9.79699 "	.44205	9.35494
2.0	9.84595 "	.46191	9.38405
2.1	9.88928 "	.47970	9.40958
2.2	9.92795 "	.49580	9.43216
2.3	9.96281 "	.51049	9.45232
2.4	9.99445 "	.52398	9.47048
2.5	0.02338	.53645	9.48694
2.6	0.04997	.54803	9.50194
2.8	.07454	.55884	9.51570
2.7	.09733	.56896	9.52838
2.9	.11857	.57848	9.54008
3.0	.13843	.58746	9.55096
3.1	.15705	.59595	9.56110
3.2	.17458	.60400	9.57058
3.3	.19111	.61166	9.57946
3.4	.20675	.61896	9.58780
3.5	.22157	.62592	9.59564
3.6	.23566	.63259	9.60306
3.7	.24905	.63897	9.61008
3.8	.26183	.64510	9.61674
3.9	.27404	.65099	9.62304
4.0	.28570	.65665	9.62904

TABLE I.—(CONTINUED).

VELOCITIES IN GUNS.

Expansions.	Slow Powders.		Quick Powders.
y	$\log Y_0$	$\log Y_1$	$\log Y + 10$
4.1	.29689	.66211	9.63476
4.2	.30761	.66738	9.64020
4.3	.31792	.67247	9.64542
4.4	.32783	.67739	9.65042
4.5	.33737	.68215	9.65522
4.6	.34656	.68676	9.65962
4.7	.35543	.69123	9.66416
4.8	.36399	.69557	9.66842
4.9	.37228	.69978	9.67246
5.0	.38030	.70388	9.67642
5.1	.38805	.70786	9.68019
5.2	.39556	.71173	9.68383
5.3	.40285	.71550	9.68735
5.4	.40993	.71918	9.69075
5.5	.41681	.72278	9.69403
5.6	.42350	.72629	9.69721
5.7	.43000	.72971	9.70029
5.8	.43632	.73305	9.70327
5.9	.44249	.73632	9.70617
6.0	.44849	.73952	9.70897
6.1	.45434	.74265	9.71169
6.2	.46005	.74571	9.71434
6.3	.46561	.74871	9.71690
6.4	.47105	.75165	9.71940
6.5	.47635	.75452	9.72183
6.6	.48153	.75733	9.72420
6.7	.48658	.76008	9.72650
6.8	.49153	.76278	9.72875
6.9	.49636	.76543	9.73093
7.0	.50110	.76803	9.73307

TABLE I.—(Continued).

Velocities in Guns.

Expansions.	Slow Powders.		Quick Powders.
y	$\log Y_0$	$\log Y_1$	$\log Y + 10$
7.1	.50574	.77059	9.73515
7.2	.51029	.77311	9.73718
7.3	.51475	.77558	9.73917
7.4	.51910	.77801	9.74109
7.5	.52339	.78040	9.74299
7.6	.52758	.78274	9.74484
7.7	.53169	.78504	9.74665
7.8	.53572	.78730	9.74842
7.9	.53966	.78952	9.75014
8.0	.54354	.79170	9.75184
8.1	.54735	.79385	9.75350
8.2	.55108	.79597	9.75511
8.3	.55476	.79806	9.75670
8.4	.55838	.80012	9.75826
8.5	.56194	.80216	9.75978
8.6	.56545	.80417	9.76128
8.7	.56889	.80615	9.76274
8.8	.57228	.80810	9.76418
8.9	.57562	.81003	9.76559
9.0	.57891	.81193	9.76698
9.1	.58213	.81380	9.76833
9.2	.58531	.81564	9.76967
9.3	.58843	.81746	9.77097
9.4	.59152	.81926	9.77226
9.5	.59457	.82104	9.77353
9.6	.59756	.82279	9.77477
9.7	.60050	.82452	9.77598
9.8	.60340	.82622	9.77718
9.9	.60626	.82790	9.77836
10.0	.60906	.82955	9.77951

TABLE I.—(CONTINUED).

VELOCITIES IN GUNS.

Expansions.	Slow Powders.		Quick Powders.
y	log Y_0	log Y_1	log $Y + 10$
10.2	.61458	.83281	9.78176
10.4	.61996	.83600	9.78396
10.6	.62520	.83912	9.78608
10.8	.63030	.84217	9.78812
11.0	.63527	.84515	9.79012
11.2	.64013	.84807	9.79206
11.4	.64485	.85092	9.79394
11.6	.64948	.85372	9.79576
11.8	.65400	.85646	9.79754
12.0	.65840	.85914	9.79926
12.2	.66272	.86178	9.80094
12.4	.66694	.86436	9.80258
12.6	.67106	.86689	9.80418
12.8	.67511	.86938	9.80572
13.0	.67906	.87182	9.80724
13.2	.68293	.87422	9.80872
13.4	.68673	.87658	9.81016
13.6	.69045	.87889	9.81156
13.8	.69409	.88116	9.81294
14.0	.69767	.88340	9.81428
14.2	.70118	.88560	9.81558
14.4	.70461	.88776	9.81684
14.6	.70799	.88989	9.81810
14.8	.71131	.89199	9.81932
15.0	.71457	.89405	9.82052
15.2	.71778	.89609	9.82170
15.4	.72093	.89809	9.82284
15.6	.72403	.90006	9.82396
15.8	.72708	.90200	9.82508
16.0	.73008	.90392	9.82616

TABLE I.—(CONTINUED).

VELOCITIES IN GUNS.

Expansions.	Slow Powders.		Quick Powders.
y	$\log Y_0$	$\log Y_1$	$\log Y + 10$
17.0	.74435	.91310	9.83125
18.0	.75757	.92168	9.83589
19.0	.76987	.92974	9.84013
20.0	.78136	.93733	9.84403
21.0	.79214	.94450	9.84764
22.0	.80230	.95130	9.85100
23.0	.81188	.95776	9.85412
24.0	.82095	.96392	9.85703
25.0	.82957	.96979	9.85978

TABLE II.

PRESSURES IN GUNS.

Expansions.	Slow Powders.			Quick Powders.	Smokeless Powders.
y	$\log X_0 + 10$	$\log X_1$	$\log X_2$	$\log X + 10$	$\log X'$
1.1	9.72174	0.12320	.21705	9.54412	9.68448—10
1.2	9.82102	.27220	.51391	9.49120	9.82203 "
1.3	9.86189	.35872	.68574	9.44254	9.89817 "
1.4	9.88089	.41969	.80663	9.39748	9.94972 "
1.5	9.88861	.46669	.89962	9.35554	9.98807 "
1.6	9.89029	.50461	.97462	9.31629	0.01829
1.7	9.88817	.53699	1.03834	9.27942	0.04299
1.8	9.88364	.56469	1.09289	9.24468	0.06379
1.9	9.87754	.58896	1.14060	9.21180	0.08164
2.0	9.87038	.61058	1.18305	9.18062	0.09725
3.0	9.78439	.75004	1.45552	8.93410	.19157
4.0	9.70563	.82896	1.60865	8.75918	.24115

TABLE II.—(CONTINUED).

PRESSURES IN GUNS.

Expansions.	Slow Powders.			Quick Powders.	Smokeless Powders.
y	$\log X_0 + 10$	$\log X_1$	$\log X_2$	$\log X + 10$	$\log X'$
5.0	9.63932	.88355	1.71414	8.62350	.27458
6.0	9.58296	.92510	1.79421	8.51264	.29978
7.0	9.53422	.95850	1.85847	8.41894	.32005
8.0	9.49138	.98633	1.91194	8.33774	.33702
9.0	9.45324	1.01017	1.95770	8.26612	.35163
10.0	9.41886	1.03096	1.99758	8.20206	.36448
11.0	9.38762	1.04947	2.03303	8.14411	.37597
12.0	9.35904	1.06591	2.06461	8.09121	.38633
13.0	9.33264	1.08088	2.09323	8.04254	.39580
14.0	9.30818	1.09456	2.11944	7.99748	.40451
15.0	9.28535	1.10713	2.14352	7.95553	.41261
16.0	9.26397	1.11878	2.16578	7.91629	.42013
17.0	9.24388	1.12959	2.18651	7.87943	.42717
18.0	9.22492	1.13972	2.20589	7.84468	.43381
19.0	9.20699	1.14922	2.22406	7.81181	.44007
20.0	9.18997	1.15815	2.24115	7.78062	.44601
21.0	9.17377	1.16658	2.25731	7.75095	.45166
22.0	9.15833	1.17458	2.27263	7.72267	.45703
23.0	9.14357	1.18216	2.28714	7.69564	.46215
24.0	9.12944	1.18940	2.30099	7.66977	.46706
25.0	9.11587	1.19630	2.31419	7.64494	.47176

TABLE III.—Density of Loading.

$\tilde{\omega}$ in lbs. C in cu. inches. $\frac{\tilde{\omega}}{C}$	Density of Loading. Δ	$\tilde{\omega}$ in lbs. C in cu. inches. $\frac{\tilde{\omega}}{C}$	Density of Loading. Δ
.010	.277	.025	.692
.011	.305	.026	.720
.012	.332	.027	.747
.013	.360	.028	.775
.014	.388	.029	.803
.015	.415	.030	.830
.016	.443	.031	.858
.017	.471	.032	.886
.018	.498	.033	.913
.019	.526	.034	.941
.020	.554	.035	.969
.021	.581	.036	.997
.022	.609	.037	1.024
.023	.637	.038	1.052
.024	.664	.039	1.080

TABLE IV.—Initial Air-Space.

$\tilde{\omega}$ in lbs. C in cu. inches. $\frac{\tilde{\omega}}{\delta C}$	Ratio of initial air-space to powder chamber. $\frac{z}{u_0}$	$\tilde{\omega}$ in lbs. C in cu. inches. $\frac{\tilde{\omega}}{\delta C}$	Ratio of initial air-space to powder chamber. $\frac{z}{u_0}$
.010	.723	.022	.391
.011	.696	.023	.363
.012	.668	.024	.336
.013	.640	.025	.308
.014	.613	.026	.280
.015	.585	.027	.253
.016	.557	.028	.225
.017	.529	.029	.197
.018	.502	.030	.170
.019	.474	.031	.142
.020	.446	.032	.114
.021	.419	.033	.087

Proportional Parts.	
1	3
2	6
3	8
4	11
5	14
6	17
7	20
8	22
9	25

TABLE V.

Area of Cross Section of Bore.

Calibre in inches.	Logarithm of Area of Cross Section in square inches.
c.	$\log \omega = \log \frac{\pi c^2}{4}$.
.315	8.89171—10
.45	9.20151—10
1.46	.22379
1.85	.42943
2.24	.59559
3.	.84933
4.	1.09921
5.	1.29303
6.	1.45139
8.	1.70127
10.	1.89509
12.	2.05345
13.	2.12298
16.	2.30333

ERRATA.

Page 9. In Example 1 insert "h equals 1.65."

Page 18. In the third equation, eleventh line, substitute $c'p$ for cp' in the second member.

Page 22. In the third line from bottom, insert a semi-colon after "atmosphere."

Page 29. In equation between (77) and (78) δ should be d.

Page 31. In the fifteenth line, the integral should be the same as in the equation before.

Page 39. In the reaction, eleventh line, the coefficient of C should be 3.

Page 42. In the sixth line from the bottom of the page, place a decima point between 1 and 8.

Page 43. In Example 1, R_2 in the value of p should read R_0.

Page 44. In the answer to Example 4, 3300 should be 2300.

Page 52. Equation (114) should have a factor g in the denominator of the second member.

Page 53. Second line from bottom of page, }
Page 54. Fourth " top " } $\dfrac{u}{u+z}$ should be $\dfrac{z}{u+z}$.

Page 54. In Equation (120) the parentheses enclosing c should enclose Δ also, so as to read $(c\Delta)^{\frac{1}{2}}$.

Page 61. In the equation on the eleventh line from the bottom of the page, the first member should be $\varphi(\gamma)$.

Page 73. The heading of Par. 69 should read MUZZLE VELOCITY FORMULA OF M. SARRAU.

Page 74. In equation (163), $\frac{1}{2}$ should be the exponent of Δ and not of k_3.

Page 75. The value of A', tenth line, should be $A\left(\dfrac{fa}{\tau}\right)^{\frac{1}{2}}$.

Page 79. The exponent of u in the denominator of the answer to Example 4 should be $\frac{5}{8}$.

Page 88. In the bottom line, change Y_2, Y_3 and Y_4 to X_0, X_1 and X_2.

Page 92. The coefficient of the last term of the answer to Example 10 should be 29.44.

Page 96. In the second equation, second line, Y should be Y_1.

Page 130. In the second line, a parenthesis should follow the fraction, and in the third line, the first equality sign should be a minus sign.

Page 140. The exponent of the second member of Equation (290) should be $\frac{1}{3}$.

Page 144. Second line from the bottom of page, the equality sign preceding 43 should be replaced by the minus sign; and in the last line, a decimal point should precede 00473.

Page 144. The first term of the bottom line should be the same as the number directly above it.